NINE
GEOMETRICALL
EXERCISES,
FOR
Young Sea-men,
And others that are ſtudious in
MATHEMATICALL PRACTICES:

Containing IX *particular* TREATISES, *whoſe* Contents *follow in the next Pages.*

All which EXERCISES are *Geometrically* performed, by a Line of *Chords* and *equal Parts*, by waies not uſually known or practiſed. Unto which the *Analogies* or *Proportions* are added, whereby they may be applied to the *Chiliads* of *Logarithms*, and *Canons* of Artificiall *Sines* and *Tangents.*

By *William Leybourn*, Philomath.

LONDON,

Printed by *J. Fleſher*, for *W. Hayes*, and are to be ſold at his Houſe at the *Croſs-daggers* in *Moor-fields* near to the *Pope's-head Tavern*, where you may have all ſorts of Mathematicall Inſtruments. 1669.

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THE CONTENTS Of the severall EXERCISES.

I.

THE *First* Exercise *contains such* Geometricall Propositions *and* Theorems *as are necessary to be known and practised for the more easie understanding of the subsequent* Exercises.

II.

The Second Exercise *contains the* Doctrine *of the* Dimension *of* Right-lined Triangles, *both* Right *and* Oblique-angled, *by* Protraction, *by a* Line *of* Chords *and* equal Parts.

III.

The Third Exercise *contains the* Doctrine *of the* Dimension *of* Sphericall Triangles, *both* Right *and* Oblique-angled, *by a* Line *of* Chords *onely*.

IV.

The Fourth Exercise *is a plain and easie* Method *of* Projecting the Sphere in Plano; *whereby the* Sides *and* Angles *of* Sphericall Triangles *are naturally laid down as they are in the* Sphere *it self. By which the* Nature *of them is discovered, and their* Sides *and* Angles *measured with* speed *and* exactness *upon the* Projection *it self.*

V.

The Fifth Exercise *declares the* Variety *of* Sphericall Problems

blems *which do naturally arise out of every* Sphericall Triangle, *both* Right *and* Oblique-angled, *and which are resolvable thereby. Described as they are perspicuous in the* Projection.

VI.

The Sixth Exercise *contains such* Astronomicall Propositions *as are of frequent Use in the Practice of* Navigation. *All which* Propositions *are resolved by measuring the* Sides *and* Angles *of* Sphericall Triangles *upon the* Projection.

VII.

The Seventh Exercise *shews how the foregoing* Astronomicall Propositions *may be applied to Practice in the Art of* Navigation: *As to find the* Latitude, Hour, Variation *of the* Compass, *&c.*

VIII.

The Eighth Exercise *contains such* Geographicall Propositions *as concern the finding of the* Distance *of* Places *upon the* Terrestrial Globe *in any Position, both by* Trigonometricall Calculation *and* Geometricall Projection.

IX.

The Ninth Exercise *shews the Use of* Right-lined Triangles *in the Practice of* Navigation; *whereby severall* Nauticall Questions *are resolved, many* Problems *in* Sailing, *both by the* Plain *and* Mercator's Chart, *performed by* Protraction, Calculation, *and wrought upon the* Chart *it self.*

These are the General Heads *of the Nine* Exercises: *All which Particulars you will find in the following* Treatises, *with addition of what is here recited: And the whole Work of all the* Exercises *performed by waies not usually known or practised.*

To

To the Reader.

HAving heretofore published to the World severall *Treatises* tending to the Practice of the *Sciences Mathematicall*; as, my *Arithmetick* in Four Parts, 1. *Vulgar*, 2. *Decimal*, 3. *Instrumental*, and 4. *Algebraicall*; my *Arithmeticall Recreations*; my *Use of the Line of Proportion made easie*; *the Use of Napiers Bones*; my *Compleat Surveyor*; my *Platform for Purchasers*, *Guide for Builders*, and *Mate for Measures*, &c. All which have found good acceptance in the World, as may appear by the severall *Editions* which some of them have passed the Press: I thought good at length to publish something that might be *usefull* and *beneficial* for my *Country-men*, *English Navigators*; hoping that I shall receive from them (for the pains I have already, and hereafter shall take for their good and advantage) no less Encouragement by their acceptance of what (out of free good will) I tender to them, then I have already from others, for what I have made publick for their Use and Service.

In the *Nine* following *Treatises* I have furnished the *Young Sea-man* with such practical *Exercises* as tend wholly to his *Employment* at *Sea*: beginning, first, with some few *Geometricall Rudiments* that are absolutely necessary for him to understand and be perfect in the Practice of, before he launch farther into the *Mathematicall Ocean*.

From these *Elements*, I proceed to the *Geometricall Dimension* of *Triangles* both *Plain* and *Sphericall*, upon the *Solution*

where-

whereof all the *Practick Parts* of the *Mathematicks* do depend. For on the *Solution* of *Right-lined Triangles*, all *Propositions* which concern *Longimetria*, or the Measuring of *Heights*, *Depths* and *Distances*, either upon the *Land* or at *Sea*; all in *Planometria*, as Measuring of *Land*, *Board*, *Glass*, *Pavements*, *Hangings*, &c. all in *Striometria*, or the Mensuration of *Solids*, such as *Stone*, *Timber*, and the like; all Problems of *Sailing*, both by the *Plain Sea-Chart*, and also by that of *Mercator*'s Projection; all that concerns *Fortification*, *Gunnery*, &c. have their dependence.—And by the *Solution* of *Sphericall Triangles*, *Problems* in *Astronomie*, *Geography*, *Sailing by the Arch of a great Circle*, *Dialling*, Measuring of the Distance of *Stars*, *Comets*, &c. are to be resolved.

Trigonometria, therefore, being the *Basis* or *Foundation* upon which the forementioned *Superstructures* are *erected*; I chose first to begin with that. And how I have performed it, and to what Uses I have applied it, which tend to the Practice of *the Art of Navigation*, the *Contents* of the severall *Exercises* will (in part) declare.

In the *Geometricall resolving* of *Triangles* both *plain* and *sphericall*, I have gone in a *Path* not before *beaten*. And therefore, if I have at any time gone out of my way, either by *deficiency* or *redundancy*, I hope the *Reader* will pardon me. I am sure the ingenious *Mathematician* (who (at first sight) will discover the manner and waies of Working, and also the Ground and Demonstration thereof) will not *carp* or *cavil* at, but rather inform *me* or the *Reader* of any such *Deviation*. But in the Pursuance of every *Exercise*, I have therein retained such a *Method*, that I could not well step aside. And for the certainty of the manner of the *Geometricall Operations*, (if they be carefully performed) I dare aver that you may (if the Radius of your Line of Chords be but 4 or 5 *Inches*) come within a few Minutes of Calculation; which will be sufficient for the *Sea-man's Use*, for him thereby to resolve such *Problems* in *Astronomie*, *Geography* or *Navigation*, as at any time he shall (in his Practice)

have

have occasion for. And that Proof or Triall may be made thereof, (though I could in a few Lines demonstrate the nature and manner of Working) I have to every *Case*, *Proposition* and *Problem* in the whole *Work* contained, added the *Analogies* or *Proportions* by which they may be *solved* by the *Canons* of *Artificial Sines* and *Tangents*, and the *Chiliads* of *Logarithms*: So that such as have those Tables (and know their Use) may work any *Proposition* by them, and by that discover the Difference between *Calculation* and this *Geometricall* way of *Operation*.

And that the *young Sea-man* may not (at any time) be to seek of *Tools* to work with, (in case of any Miscarriage at Sea) I have not onely shewed him how to *use*, but also the manner how to *make*, all such *Instruments* as will be in use to him in the Practice of these *Exercises*: As of his *Line of Chords*, which is the chief; and also of a *Sea-Chart* according to *Mercator*'s Projection, without the help of a *Table* of *Meridionall Parts*, or a *Meridian Line*, but *easily* and *speedily* by a *Line of Chords* onely; and that either *General*, from the *Æquinoctial* towards either *Pole*; or *Particular*, for any designed *Navigation*.

Moreover, in the Working of the severall *Questions* by *Mercator*'s *Chart*; I have, at the end of every *Problem*, given an account of the Difference arising between Working by that and the *Plain Sea-Chart*; whereby the *Errours* of the one are clearly detected, and the *Verity* of the other discovered.

I might have proceeded farther, to the performance of other things by the *Line of Chords* onely; as to shew the manner of *Sailing* by the *Arch of a great Circle*: but that, with some other things tending to *Navigation*, I shall shortly make publick in a *Treatise* formerly written by my self and Mr. *Vincent Wing*, now in my hands, and almost ready for the *Press*, and wholly designed for the Use of *Sea-men*.—And for farther Use of this Line, I have in a Treatise (now in the Press) taught the manner how to delineate all manner of *Sun-Dials* by *projecting* of the *Sphere in Plano*, whereby not onely the *Making*, but the *Reason* also, of

Dials is discovered. To which Treatise (if my present occasions will permit) I shall adde both an Arithmeticall and an Instrumentall way of *Dialling* also.

Thus, friendly Reader, at present, I present thee with This; and in a few Months thou shalt (God willing) participate of those above mentioned. All which I hope will be as kindly accepted, as they are freely tendered by

Will. Leybourn.

Arts

Arts and Sciences MATHEMATICALL Professed and Taught by the Authour.

Arithmetick,
- In Whole Numbers, and Fractions.
- In Decimals, and by Logarithms.
- Instrumentally, by Decimal Scales, Napiers Bones: and to extract the Square and Cube Roots by Inspection.

Geometrie:
- The Principles thereof with the Practice, and Demonstration.

Astronomie:
- The Description of the Circles of the Sphere.
- The Use of the Globes, Celestiall, and Terrestriall.
- To project the Sphere *in Plano* upon any Circle, Right, or Oblique.

And upon these Foundations the following Superstructures.

The Use of Geometricall Instruments, in the Practice of
- *Longimetria*, or the Mensuration of Heights, Depths, Distances, as of Trees, Towers, &c. Mines, Wells, Descents, &c. Churches, Towers, &c.
- *Planometria*, or the Mensuration of Board, Glass, Pavement, Tiling, &c. Or any other Superficies.
- *Stereometria*, or the Mensuration of Timber, growing or squared. Stone, regular or irregular. Cask, commonly called Gageing.
- *Geodæsia*, or the Measuring of Land divers waies, and by severall Instruments; to draw the Plot of a whole Mannor or Lordship; to cast up the Content thereof; and to beautifie the same with all necessary Ornaments thereunto belonging.

Trigonometria:
- Or, the Mensuration of Triangles, both Plain, and Sphericall.
- The Application thereof, in the solution of Problems in Geometry. Astronomie. Geographie. Navigation. Fortification. Dialling, &c.

Navigation:
- The Principles thereof, and the manner of Sailing by The Plain Sea-Chart. *Mercator*'s Chart. The Arch of a great Circle.

Horologiographia Or Dialling:
- *Arithmetically*, by the Tables of Sines. Tangents. Logarithms.
- *Geometrically*, by Scale, and Compasses.
- *Instrumentally*, by the Sector, Quadrants, Scales, and other Instruments accommodated with Lines for that purpose.

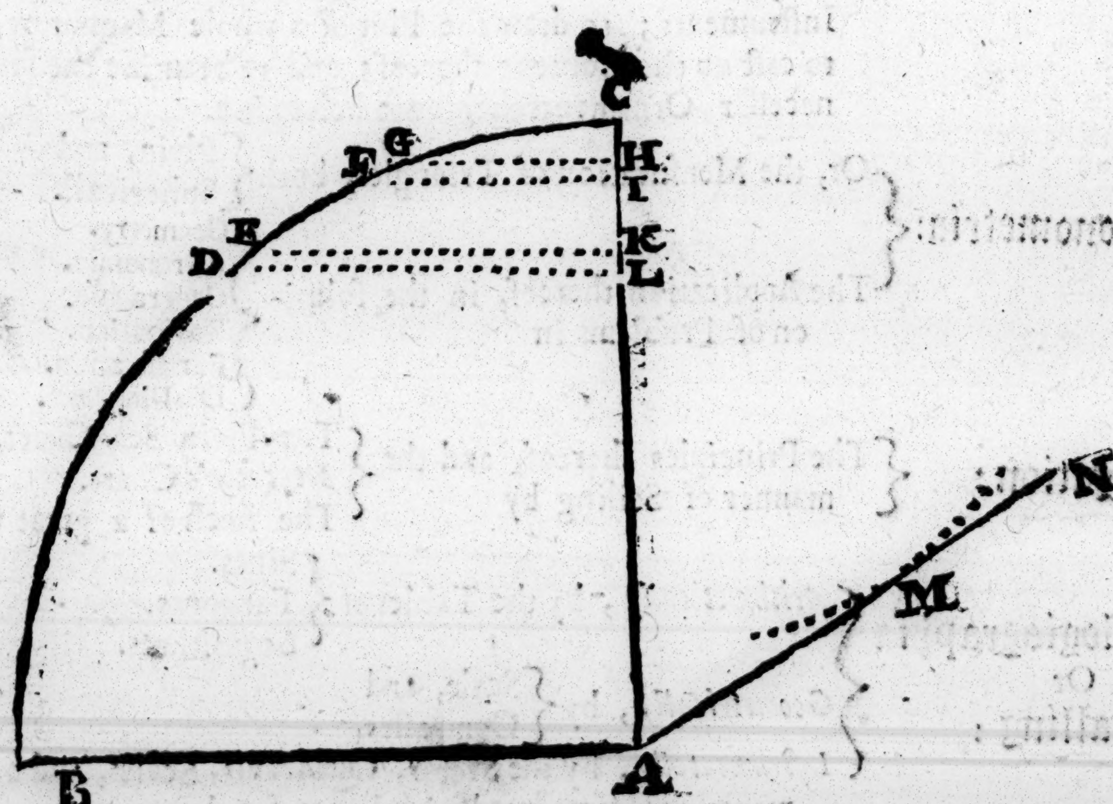
You may hear of him at Mr. *Hayes*'s at the *Cross-daggers* in *Moor-fields*.

ADVERTISEMENT.

AND here I thought good to give notice, That if any Gentlemen studious in the Mathematicks have or shall have occasion for Instruments thereunto belonging, or Books to shew the Use of them, they may be furnished with all sorts usefull both for Sea or Land, either in Silver, Brass or Wood, by Walter Hayes, at the Cross-daggers in Moor-fields, next door to the Pope's-head Tavern; where they may have all sorts of Maps, Globes, Sea-plats, Carpenters Rules, Post and Pocket-Dials for any Latitude.

This Scheme having relation to the Fourth and Fifth Cases of Sphericall Triangles in Page 53 and 54, being casually omitted, is here inserted.

alexandre hamilton



GEO-

GEOMETRICALL PROPOSITIONS and THEOREMS,

Necessary to be known and practised for the more easie understanding of the subsequent EXERCISES.

The First EXERCISE.

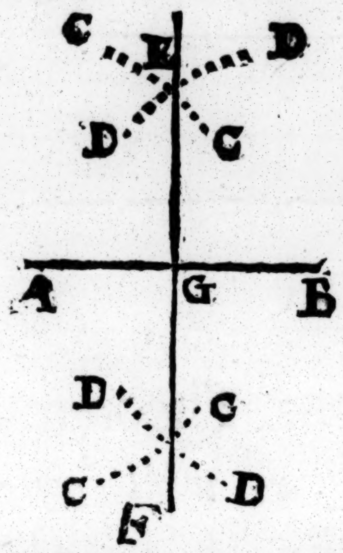
THE ARGUMENT.

THE *following* Propositions, Theorems *and* Problems, *are such as will come in continuall use in the Practice of the subsequent* EXERCISES, *and therefore I have here promiscuously inserted such as do relate to the following* Treatises, *and ought throughly to be* understood *and often* practised; *by which means nothing in the ensuing* Cases, Propositions *and* Problems, *will be difficult to be understood, but that you may gradually proceed, in your* Practice, *from* Exercise *to* Exercise, *without being referred to any other* Book *or* Books.

GEOMETRICALL PROPOSITIONS.

PROP. I.

A right Line *being given, to divide the ſame into two* equal *parts at* right Angles.



LET the right Line, to be divided, be A B.—First, open your Compaſſes to any diſtance greater then the length of half the given Line; then ſetting one foot in A, with the other foot deſcribe the Arches C C, both above and below the Line A B.—Secondly, (the Compaſſes ſtill reſting at the ſame diſtance,) ſet one foot in B, and with the other deſcribe the Arches D D, cutting the former Arches C C in the Point E above, and F below the given Line A B.—Laſtly, draw the Line E F, which will divide the given Line A B in two equal parts, in the Point G, and at right Angles.

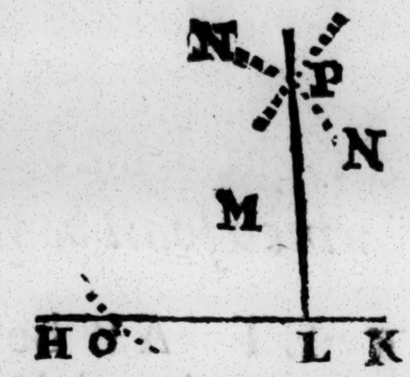
PROP. II.

Upon a right Line *given, to erect a* Perpendicular *upon any part thereof.*

LET the given Line be H K, and from the Point L let it be required to erect a Perpendicular.—First, open your Compaſſes to any convenient diſtance, and ſetting one foot in the given Point L, with the other make a Mark or

Point

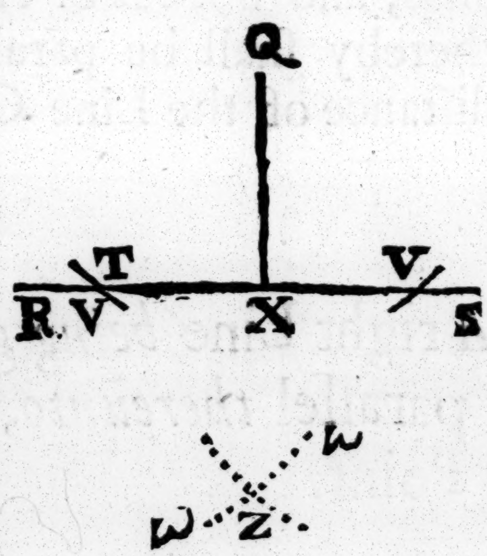
Point at pleasure, as M. Then keeping the Compass-point in M, with the other describe the Arch N N above the Point L, and also another Arch at O, cutting the given Line in O.——Lastly, lay a Ruler from O to M, which will cut the Arch N N before drawn in the Point P. So a Line drawn from P to L shall be a Perpendicular to the given Line H K, and from the Point L.



Prop. III.

From a Point *above, to let fall a* Perpendicular *upon a* right Line *given.*

LET the Point given be Q, and the Line upon which the Perpendicular is to fall be R S.——First, opening the Compasses to any distance greater then Q X, set one foot in Q, and with the other cross the given Line R S in the Points T and V.—Secondly, (the Compasses unaltered,) set one foot in T, and with the other describe the Arch *w w*, below the given Line R S.——Thirdly, remove the Compasses to V, (being still at the same distance) and cross the Arch *w w* in the Point Z—. Lastly, a Ruler laid from Q to Z will cut the Line R S in the Point X. So a Line drawn from Q to X shall be perpendicular to the given Line R S.



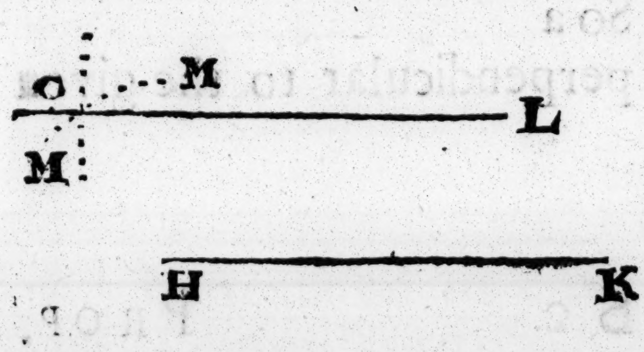
Prop. IV.

A right Line *being given, to draw another* right Line *which shall be* parallel *thereto at any distance required.*

LET AB be a Line given, and let it be required to draw another right Line which shall be parallel thereunto, and at the distance of the length of the Line C.——First, take the length of the Line C in your Compasses, and setting one foot towards one end of the given Line, as at D, describe the Arch E.——Secondly, set one foot of the Compasses towards the other end of the given Line, as at F, and describe the Arch G.——Lastly, lay a Ruler to the Arches E and G, so that the Ruler onely touch the Arches, and not cut or cross them in any part. So a Line drawn thereby shall be parallel to the given Line AB, and at the distance of the Line C.

Prop. V.

A right Line *being given, to draw another* right Line parallel *thereunto, which shall pass through a given* Point.



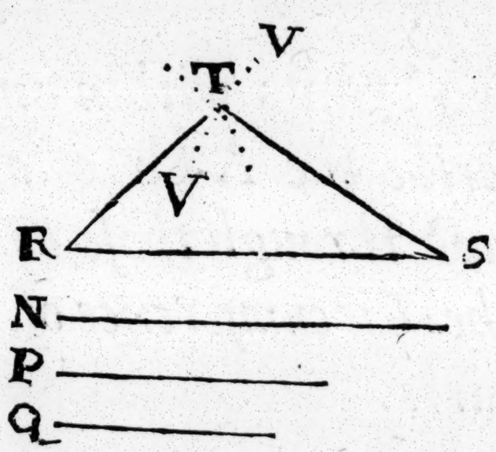
LET the given Line be HK, to which let it be required to draw a parallel Line, which shall pass through the Point L.——First, take in your Compasses the distance KL, and setting one foot of the Compasses in H, with

H, with the other describe the Arch M M.——Secondly, take the Line HK in your Compasses, and setting one foot in the given Point L, with the other cross the Arch MM in the Point O. So a Line drawn from L to O shall be parallel to the given Line H K, and shall pass through the Point L.

PROP. VI.

Three right Lines *being given, to make a* Triangle, *whose* three Sides *shall be equal to the* three *given* Lines.

LET the three Lines given be N, P, Q.——First, take the Line N in your Compasses, and lay that down from R to S.——Secondly, take the Line P in your Compasses, and setting one foot in S, with the other describe the Arch V V.——Thirdly, take the Line Q in your Compasses, and setting one foot in R, with the other cross the Arch V V, in the Point T.——Lastly, draw the Lines T R, and T S. So shall you have constituted the Triangle TRS, whose three Sides are equal to the three given Lines, N, P, Q.

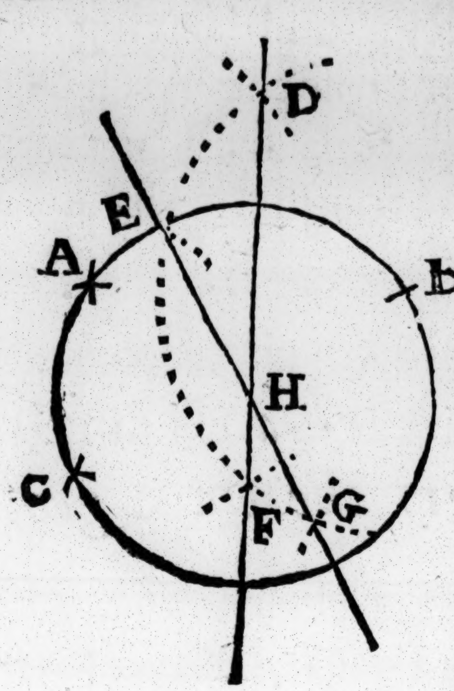


PROP. VII.

Three Points *(which lie not in a straight Line) being given, to finde the* Centre *of a* Circle, *which being described shall pass through the three given* Points.

LET the three given Points be A, B, C.——First, open your Compasses to any distance greater then half the distance between A and B: and setting one foot in B,

with the other describe the Arch G D. Then remove the

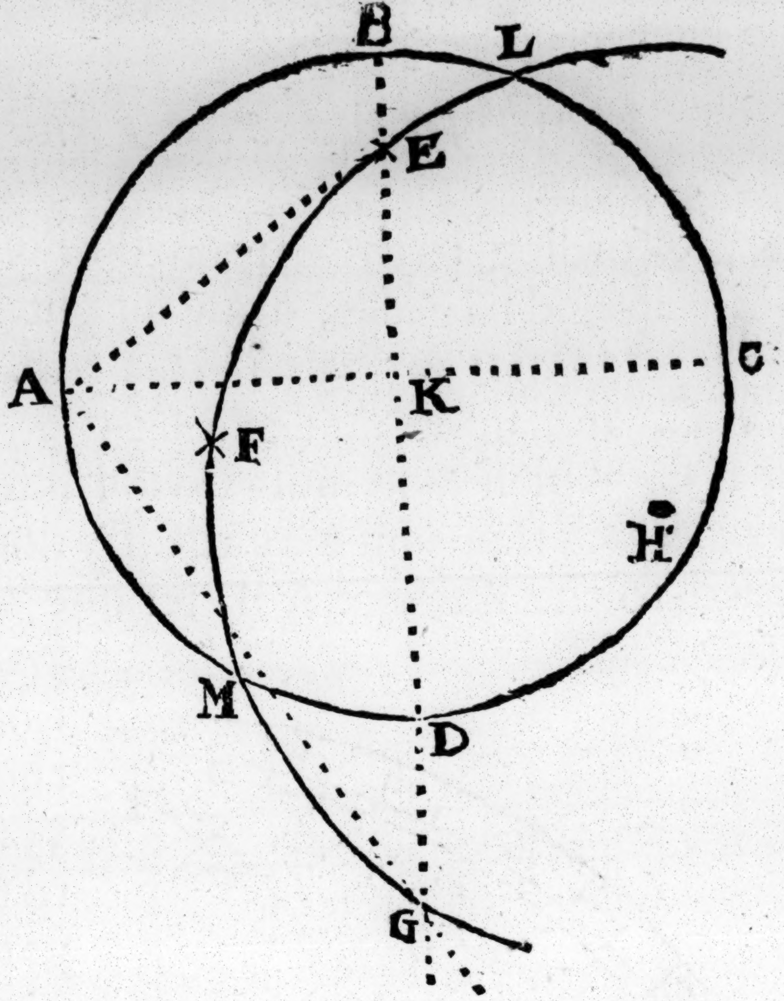


Compasses, set one foot in A, and with the other cross the former Arch in the Points D and F, and draw the Line D F.———Secondly, (the Compasses still continuing at the same distance,) set one foot in the Point C, and with the other cross the Arch (before drawn) in the Points E and G, and draw the Line E G, crossing the other Line D F in the Point H. So shall H be the Centre of the Circle, which being described shall pass directly through the three given Points, A, B, C.

Prop. VIII.

Two Points *within any* Circle *being given, how to describe the* Arch *of another* great Circle *which shall pass through those two given* Points, *and also divide the Circumference of the given* Circle *into two equal parts.*

LET the two Points given be E and F, within the Circle A B C D.———First, through either of them (as through E) draw the right Line E D, passing through the Centre of the Circle at K.———Secondly, draw the Line A C at right Angles to B D; so shall the Circle be divided into four equal parts or Quadrants, by the Lines A C and B D. ———Thirdly, draw the Line E A, and upon the Point A (by the II. Prop.) erect the Perpendicular A G, cutting the Line B D (it being extended) in the Point G; so have you three Points, E, F, and G, through which (by the last Prop.) ou may draw a Circle to pass, whose Centre will be at H: upon



upon which Point if you describe an Arch of a Circle, at the distance HE or HF, it will pass through the two given Points E and F, and divide the Circle ABCD into two equal parts, in the Points L and M, which was required. And that this Arch, thus drawn, doth divide the Circle into two equal parts, is evident, for a Line drawn from L to M will pass directly through the Centre K.

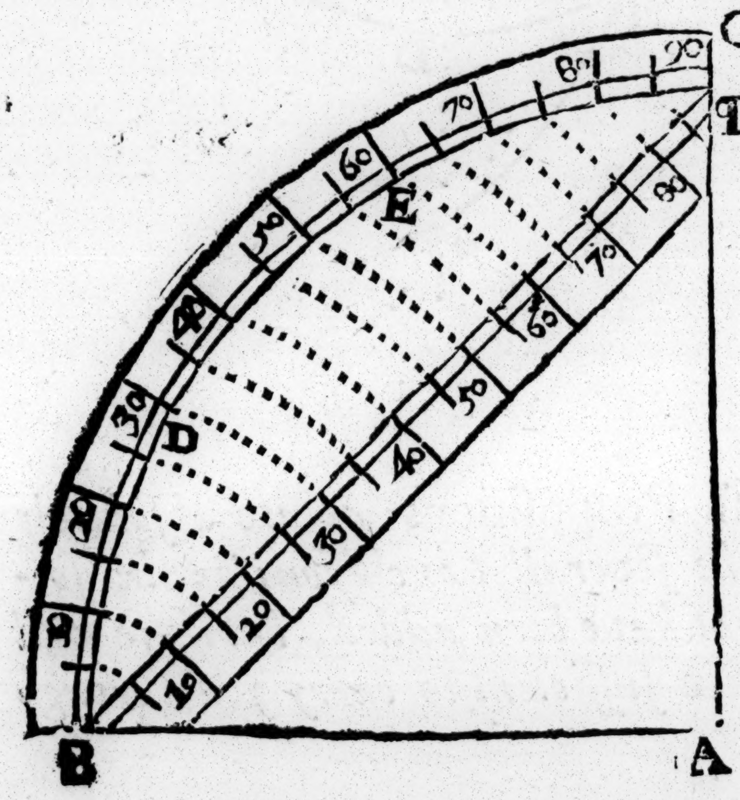
These are such Geometricall Propositions *as are absolutely necessary for the working of the severall Conclusions in the following* Exercises. *More might have been added, but these well understood and practised will be sufficient to carry you through this Work.*

And now I will describe unto you the making of the slight Instrument by which all contained in this Book is performed, namely, The Line of Chords, *and shew you the general Use thereof, in the* protracting *or* laying down *of* Angles *of any quantity: Or if any* Angle *be already* laid down, *to finde (thereby) the quantity thereof.——Then will I give you some few usefull and necessary* Theorems, *chiefly appertaining to* Trigonometry, *or the* Solution of Triangles, *and so conclude this first* EXERCISE.

How

How to make a Line of Chords.

ACcording to the largeneſs of your *Line of Chords* you intend to make, draw a right Line, as A B, and upon the Point A (by the II. Prop.) erect the Perpendicular A C, and upon A (as a Centre) deſcribe the *Quadrant* B D E C, which you muſt divide into 90 equal parts or *Degrees*. Which that you may readily doe, your Compaſſes being opened to the diſtance A B, ſet one foot in B, and the other will reach to E; alſo ſet one foot in C, and the other will reach to D: ſo is your *Quadrant* divided into three equal parts, each part containing 30 degr. This done, divide each of theſe three parts into three more; ſo ſhall you have divided your *Quadrant* into 9 equal parts, each containing 10 degr. and each of theſe 9 parts, being divided into halves, will contain 5 degr. and (if you make your Line large enough) you muſt divide thoſe into 5 equal parts, which you may very well doe, if the Line A B be but two inches long, as all the *Schemes* and *Figures* in theſe *Exerciſes* are drawn by a *Line of Chords* of that length.



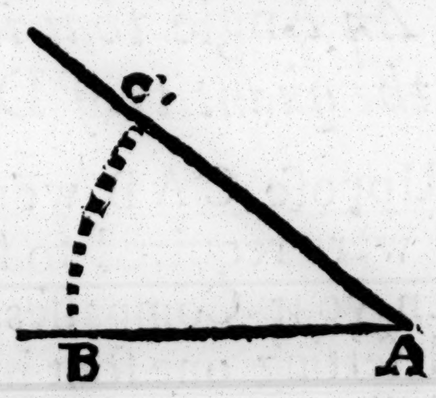
Your *Quadrant* being thus divided into 90 degr. draw the Line B C, and parallel thereto two other Lines, one pretty cloſe to B C, to contain the ſmall Diviſions, and the other at a larger diſtance, to ſet the Figures in. Now it is the Line B C which is called the *Line of Chords*, (poſſibly for this Reaſon, the

the Arch or Ark B D E C repreſenting *Arcus*, a *Bow*, and B C the *String* or *Chord* thereof) the diviſions whereof are to be transferred from the degrees of the *Quadrant* B D E C, in this manner.——Firſt, ſetting one foot of your Compaſſes in B, extend the other to 80 degr. in the *Quadrant*, and from the diviſion of 80 degr. in the *Quadrant* draw the Arch 80, 80, which will cut the *Chord-Line* in 80; doe ſo with 70, 60, 50, *&c.* and the like with every fifth degree, as you ſee in the Figure. And if your Line be very large, you may doe ſo to every ſingle degree, and part of a degree. And by this means have you reduced the degrees of the *Quadrant* B D E C to the ſtraight Line B C, more commodious to be ſet upon a Ruler, then the crooked Arch B D E C.

The Uſes *of the* Line of Chords.

THE *Vſes* of this Line are principally two. The *one* is, To *protract* or *lay down* upon Paper an *Angle* of any quantity (that is, of any number of *degrees*) required.——The *other Vſe* is, If an *Angle* be already *protracted* or *laid down*, to finde how many *degrees* and parts of a *degree* it containeth.——In both which I would have the Reader very perfect, becauſe very much contained in this Book hath dependence thereupon.

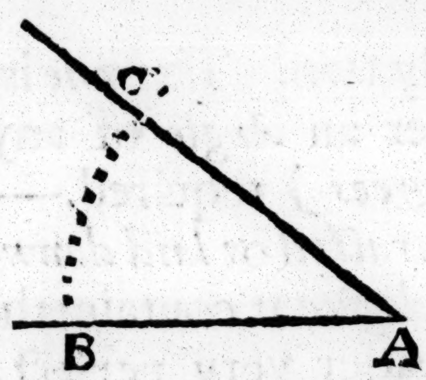
And here it will be neceſſary that I give you the Definition of an Angle. Know therefore that an Angle is the Inclination or bowing of two right Lines the one to the other.———As the two right Lines C A and B A incline the one to the other, and touch or meet each other in the Point A, in which Point, by reaſon of the inclination of the ſaid Lines, is made the Angle C A B.



And here note that an Angle is commonly ſigned by three Letters, the middlemoſt whereof ſignifies the angular Point. As in this Figure, when we ſay the Angle C A B, you are to underſtand the very Point at A.

I. *How to* protract *(or lay down upon Paper) an* Angle *containing any number of* Degrees *and* Minutes *by the* Line of Chords.

DRaw a right Line at pleaſure, as A B, and from the Point A let it be required to protract or lay down an Angle containing 40 degrees.——Firſt, open your Compaſſes alwaies to 60 degr. of your *Line of Chords*, (which is equal to the Line A C of the *Quadrant*,) and with this diſtance, ſetting one foot of the Compaſſes upon the Point A, with the other foot deſcribe the Arch B C.——Secondly, take in your Compaſſes 40 degr. (which is the quantity of the Angle to be laid down) out of the *Line of Chords*, from the beginning thereof, and ſetting one foot in B, the other will reach to C upon the Arch: wherefore through the Point C draw the Line C A. So ſhall the Angle at A contain 40 degr. as was required.



II. *An* Angle *that is already* protracted, *how to finde the quantity of* Degrees *it containeth.*

SUppoſe C A B were an Angle already *protracted*, and it were required to finde the quantity thereof.——Firſt, open your Compaſſes to 60 degr. of your *Line of Chords*, and ſetting one foot in A, (the angular Point) with the other deſcribe the Arch B C.——Secondly, take in your Compaſſes

paſſes the diſtance between B and C, which diſtance apply to your *Line of Chords*, (by ſetting one foot in the beginning thereof) and you ſhall finde the other to fall upon 40 degr. which is the quantity of the Angle at A.

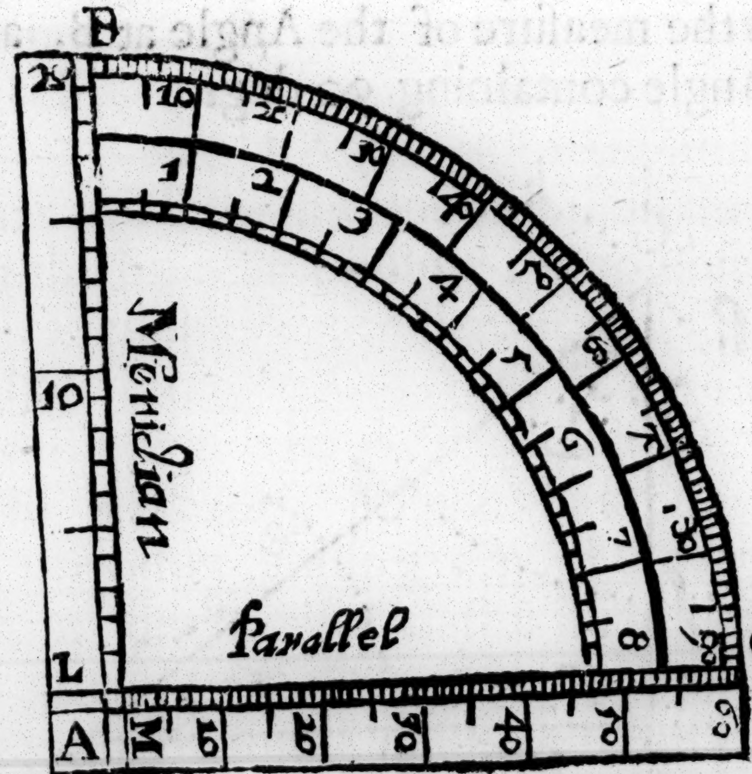
Thus have you the *Uſes* of your *Line of Chords* in *protracting* and finding the quantities of *Angles*. And now it will not be impertinent, if in this place I ſhew you how *Angles* may be *protracted* and laid down, and alſo their quantities found, by an *Inſtrument* which I ſhall make uſe of towards the end of this *Book*, which I call a *Protracting Quadrant*.

Its Deſcription.

IT is no other then a *Quadrant* made upon a piece of very thin Braſs, and divided into 90 degr. the Braſs being cut away cloſe to the diviſions of the degrees on the out-ſide, and alſo the hollow within, ſo that there remains nothing but the *Limb* and the two *Sides*, as you may diſcern by the Figure. In the *protracting* or laying down of *Angles*, and in finding of the quantity of *Angles* already laid down, this is

Its Uſe.

SUppoſe you were to finde the quantity of the Angle C A B. Hold a *Pin* or *Needle* upon the angular Point at A, to which bring the Centre of your *Quadrant*, (noted alſo with A) and there turn it about, till the *Meridian Line* thereof, A B, lie upon the Line A B of the *Angle*: then ſee under what degrees of the *Quadrant* the Line A C



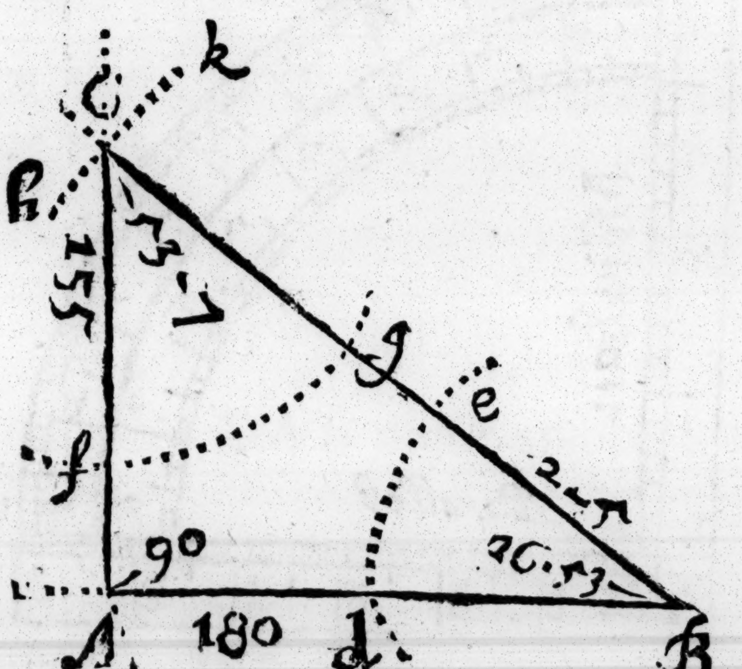
lieth, which you shall find to lie just under 40 degr. And such is the quantity of the Angle C A B.—And if upon the Line B A you were to protract such an Angle of 40 degr. Lay the *Meridian* Line of the *Quadrant* upon the Line A B, (the Centre of the *Quadrant* upon the Point A) and with your Needle make a prick or point just against 40 degr. of the *Quadrant's* limb. So a Line drawn from A through this Point shall make an Angle of 40 degr.

Trigonometricall Theorems.

1. A Triangle is a Figure consisting of three Sides, and as many Angles; as is the Figure C A B.

2. Any two Sides of a Triangle are called the Sides of the Angle contained by them; as the Sides C B, and A B, are the Sides containing the Angle C B A.

3. The measure of an Angle is the quantity of the Arch of a Circle, described upon the angular Point, and cutting both the Sides containing the Angle. As in the Triangle A C B, the Arch *f g* is the measure of the Angle at C, the Arch *d e* is the measure of the Angle at B, and the Angle at A is a right Angle containing 90 degr.



4. A Degree is the 360. part of any Circle. Therefore,

5. A Semicircle contains 180 degr. And

6. A Quadrant (or right Angle) contains 90 degr.

7. The Complement of an Angle less then 90 degr. is so much as that Angle wanteth of 90 deg.

As

As the Angle A C B of the Triangle containeth 53 degr. 7. min. the Complement whereof is 36 degr. 53 min. which is ſo much as 53 degr. 7 min. wanteth of 90 degr. For if you ſubtract 53 degr. 37 min. from 90 degr. (or from 89 degr. 60 min. for eaſe in ſubtracting) the remainder will be 36 degr. 53 min.

8. The Complement of an Angle to a Semicircle is ſo much as that Angle wanteth of 180 degr. So the Angle C being 53 degr. 7 min. take 53 degr. 7 min. from 180 degr. (or from 179 degr. 60 min.) and the remainder will be 126 degr. 53 min. which is the Complement of the Angle C to 180 degr.

9. An Angle is either right, acute, or obtuſe.

10. A Right Angle is that whoſe meaſure is 90 degr. or a Quadrant.

11. An Acute Angle is leſs then a right Angle, and alwaies contains leſs then 90 degr.

12. An Obtuſe Angle is greater then a right Angle, and alwaies contains more then 90 degr.

13. A Triangle is either right-angled, or oblique-angled.

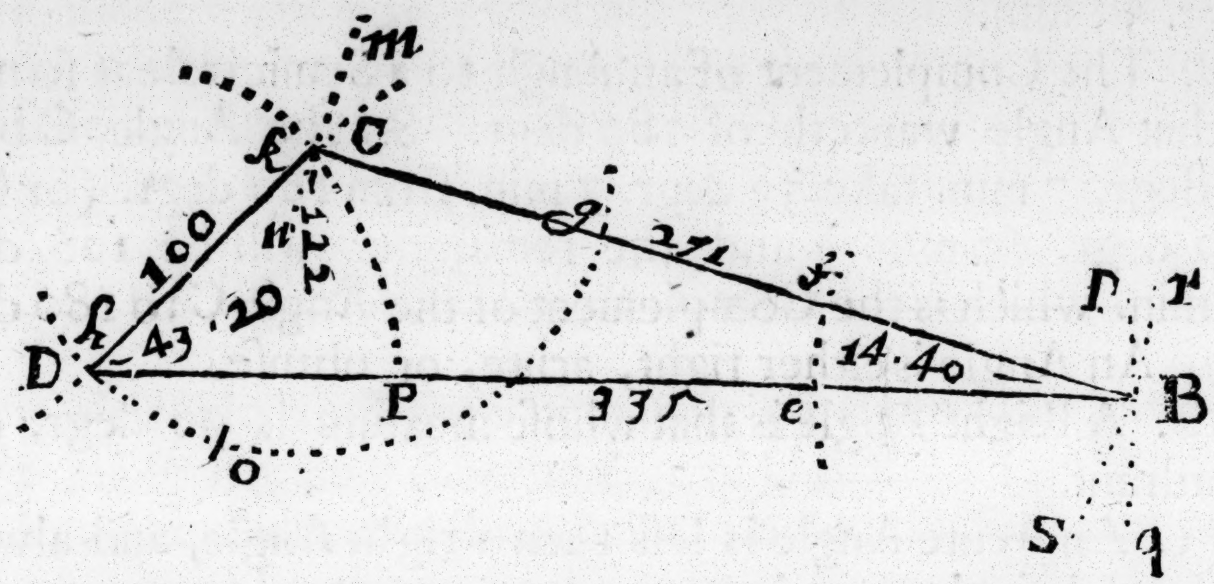
14. A right-angled Triangle is ſuch a Triangle as hath one right Angle. As the Triangle C A B hath one right Angle, namely, that at A, which containeth juſt 90 degr.

15. In every right-angled Triangle, that Side which ſubtendeth (or lieth oppoſite to) the right Angle is called the *Hypotenuſe*; and of the other two Sides the one is called the Perpendicular, and the other the Baſe, at pleaſure: But moſt commonly the ſhorter ſide is called the Perpendicular, and the longer the Baſe. Thus in the Triangle C B A, B C is the Hypotenuſe, C A the Perpendicular, and A B the Baſe.

16. In every right-angled Triangle, if you have one of the acute Angles given, the other is alſo given, it being the Complement thereof to 90 degr. As in the Triangle C A B, if you have the Angle at C 53 degr. 7 min. given, you have alſo the Angle at B given, it being the Complement of that at C to 90 degr. wherefore take 53 degr. 7 min. from

90 degr. and there will remain 36 degr. 53 min. which is the quantity of the Angle at B.

17. In all right-lined Triangles whatsoever, (either right-angled or oblique-angled) the three Angles together are



equal to two right Angles, or contain 180 degr. Therefore, if you have any two Angles of a Triangle given, you have also the third given, it being the Complement of the other two to 180 degr.——Thus in the Triangle C D B, if there were given the Angle C D B 43 degr. 20 min. and the Angle C B D 14 degr. 40 min. I say, by consequence you have the third Angle D C B also given, it being the Complement of the other two to 180 degr. For the two given Angles B D C 43 degr. 20 min. and C B D 14 degr. 40 min. being added together, make 58 degr. which being taken from 180 degr. there will remain 122 degr. the quantity of the obtuse Angle D C B.

18. In all Triangles whatsoever, the Sides are in proportion one to the other as the Sines of the Angles opposite to those Sides. So in the Triangle C D B, the Sine of the Angle at D is the proportion to the Side C B, which is opposite to it; as the Sine of the Angle at B is to the Side C D, or the Angle at C to the Side D B.

THE

THE *SOLUTION* Of Right-lined TRIANGLES By the *LINES* of *EQUAL PARTS, & CHORDS.*

The Second EXERCISE.

WHAT *a* Line of equal parts, *and a* Line of Chords, *is, is not unknown to any conversant in the least with* Mathematicall *Practices. For a* Line of equal parts *is onely a Line of any length equally divided into any number of equal parts, as* 10, 20, 30, 60, 100, 1000, &c.——*And a* Line of Chords *is no other then the Degrees of a Quadrant, or quarter of a Circle, (which contains* 90 *degr.) transferred from those degrees to a straight Line; as in the foregoing Exercise I have shewed. These Lines*

Lines of equal parts *and* Chords, *being put upon a plain* Ruler, (*which any that make* Mathematicall Instruments *know how to doe*) *will, by the* Precepts *following, measure all manner of* Triangles, *whether* plain *or* spherical. *But before I come to the Solution of the severall* Cases *appertaining to* Triangles, *I would have you take notice, That every* Triangle, *whether* plain *or* sphericall, *consisteth of six parts, namely, of three* Sides, *and as many* Angles; *any three of which being given, a fourth may be found. Now in the resolving of* right-lined Triangles, *if they be* right-angled, *I call the Side opposite to the* right Angle *the* Hypotenuse; *and of the other Sides, comprehending the* right Angle, *I call the longer of them the* Base, *and the shorter, the* Perpendicular.——*But in the Solution of* oblique-angled plain Triangles, *I call the longest Side of the* Triangle *the* Base; *and the other two, the two* Sides, *without any other distinction of Denomination or Name. These things being premised, I come now to the Solution of* plain Triangles, *both* right *and* oblique-angled. *And*

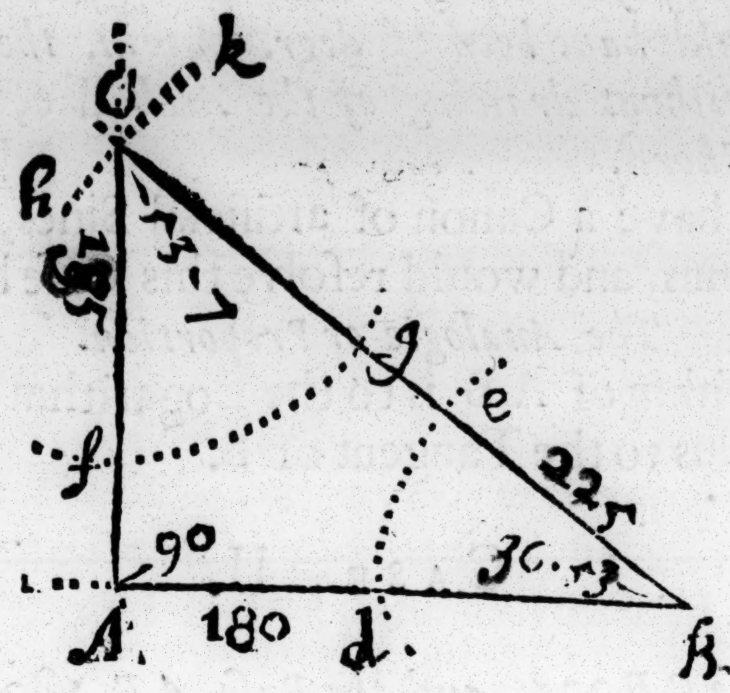
I. *Of Right-angled plain Triangles.*

THE Triangle which I shall make use of in the severall Cases belonging to a *right-angled plain Triangle* shall be this following, C A B, in which

		parts.
A B, the Base,	contains	180
C A, the Perpendicular,	contains	135
C B, the Hypotenuse,	contains	225

And		deg. — m.
A, the right Angle,	contains	90—00
C, the Angle at the Per.	contains	53—07
B, the Angle at the Base,	contains	36—53

CASE



CASE I.

The Base B A 180, and the Perpendicular C A 135, being given, to finde the Angles B and C.

DRaw a Line A B, and from your Scale of equal parts take 180, and set them from A to B; then on the Point A raise the Perpendicular A C, and because it contains 135, take 135 parts from your Scale of equal parts, and set them from A to C; then draw the Line C B: which three Lines will constitute the Triangle C A B.

Now to finde the Angles C and B, take in your Compasses 60 degr. of your Line of Chords, and setting one foot in C, describe the Arch *f g*; also setting one foot in B, describe the Arch *d e*: then take in your Compasses the distance from *f* to *g*, which measured upon your Line of Chords will reach from the beginning thereof to 53 degr. 7 min. and such is the quantity of the Angle at C.——In like manner take the distance between *d* and *e* in your Compasses, that distance applied to your Line of Chords will reach to 36 degr. 53 min. ——*Or, when you had found the quantity of the Angle C to be*

53 degr. 7 min. if you had subtracted that from 180 *degr. the remainder would have been* 36 *degr.* 53 *min. the quantity of the Angle at B, without drawing of the Arch* d e, *and measuring it upon your Chord.*

For such as have a Canon of artificial Sines, Tangents and Logarithms, and would resolve this Case by them, this is

The Analogie or Proportion.

As the Logarithm of A B is to the Logarithm of A C,
So is the Radius to the Tangent of B.

Case II.

The Hypotenuse C B 225, *and the Base A B* 180, *being given, to finde the Angles B and C.*

DRaw a right Line A B containing 180 parts of your Scale of equal parts, also out of the same Scale of equal parts take 225, your Hypotenuse, and setting one foot of your Compasses in B, with the other describe the obscure Arch *h k*; then on the Point A raise the Perpendicular A C, which will cut the obscure Arch *h k* in C; then draw the Line C B, so have you the Triangle C A B: then may you measure the quantity of the Angles at C and B as in the last *Case*. And so will C be 53 degr. 7 min. and B 36 degr. 53 min.

The Analogie or Proportion is,

As the Log. of C B is to the Radius,
So is the Log. of the Side A B to the Sine of C.

Case III.

The Base A B 180, *the Angle C* 53 *degr.* 7 *min. and the Angle B* 36 *deg.*53 *min. being given, to finde the Perpendicular C A.*

DRaw a right Line A B containing 180 parts of your Scale, for the Base of your Triangle; then taking 60 degr. from your Line of Chords, on the Point B describe the Arch

d e,

d e, and (because the Angle at B contains 36 degr. 53 min.) take 36 degr. 53 min. from your Chord, and set it from *d* to *e*, and from B, through the Point *e*, draw the Line B C. Also upon the Point A erect the Perpendicular A C, crossing the Line B C in C. So have you formed the Triangle C A B. Lastly, take the length of the Line A C in your Compasses, and measuring it upon your Line of equal parts, you shall find it to contain 135. And that is the length of the Perpendicular C A.

The Analogie or Proportion is,

As the Sine of the Angle at C is to the Log. of A B,
So is the Sine of the Angle B to the Logar. of C A.

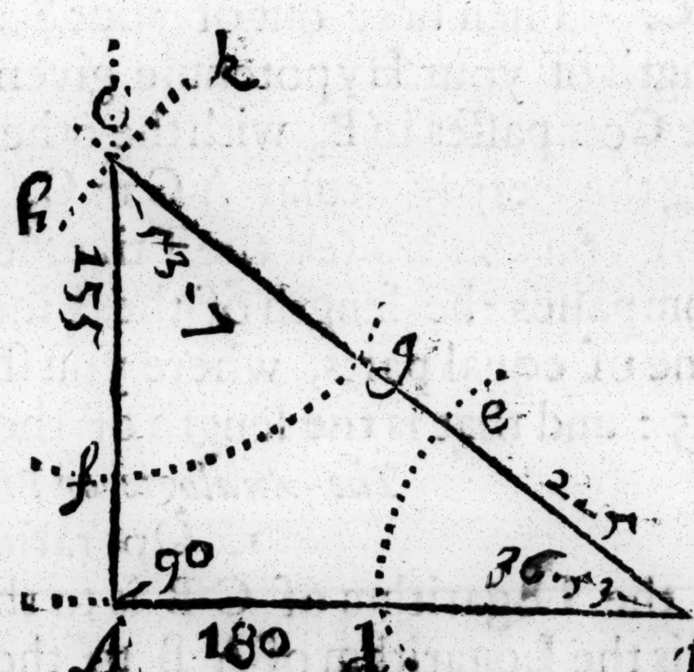
Or,

As the Radius is to the Logar. of A B,
So is the Tangent of B to the Logar. of C A.

CASE IV.

The Hypotenuse C B 225, the Angle C 53 degr. 7 min. and the Angle at B 36 degr. 53 min. given, to finde the Base B A, and the Perpendicular C A.

DRaw a right Line C B containing 225 of your Line of equal parts; then taking 60 deg. out of your Line of Chords, set one foot of the Compasses in B, and with the other describe the Arch *e d*; also (the Compasses continuing at the same distance) place one foot in C, and with the other describe the



Arch *g f*. Then from the Point B, and through the Point *d*, draw a right Line; also from the Point C, and through the Point *f*, draw another right Line: these two Lines will intersect or cross each other in the Point A, forming the Triangle C A B. Lastly, take the Line A B in your Compasses, and applying it to your Scale of equal parts, you shall finde it to contain 180; and that is the length of the Base A B. Likewise A C being taken in the Compasses, and measured upon the Line of equal parts, will be found to contain 135, which is the length of the Perpendicular C A.

The Analogie or Proportion is,

As the Radius is to the Logarithm of C B,
So is the Sine of C to the Logarithm of A B,
And the Sine of B to the Logarithm of C A.

Case V.

The Hypotenuse C B 225, *and the Base A B* 180, *being given, to finde the Perpendicular C A.*

DRaw a right Line A B containing 180 of your Scale of equal parts, and upon the end A erect a Perpendicular A C. Then take out of your Scale of equal parts 225, (the length of your Hypotenuse given,) and setting one foot of the Compasses in B, with the other describe the Arch *h k*, cutting the Perpendicular A C in C, then draw the Line C B: so have you constituted the Triangle C A B. Lastly, take in your Compasses the length of the Line A C, and apply it to your Line of equal parts, where you shall finde that it will contain 135: and that is the length of the Perpendicular C A.

The Analogie or Proportion is,

1. Operation.

As the Logarithm of C B is to the Radius,
So is the Logarithm of A B to the Sine of C.

2. Opera-

2. Operation.

As the Radius is to the Logarithm of C B,
So is the Sine of B (the Complement of C) to the Log. of C A.

Case VI.

The Base A B 180, *the Angle C* 53 *degr.* 7 *min. and the Angle B* 36 *degr.* 53 *min. being given, to finde the Hypotenuse C B.*

DRaw a right Line A B containing 180 parts of your Scale, for the Base of your Triangle, and on the end A erect a Perpendicular A C. Then take 60 degr. out of your Line of Chords, and upon the Point B, with that distance, describe the Arch *d e*; and (because the Angle at B is 36 degr. 53 min.) take 36 degr. 53 min. from your Line of Chords, and set it upon the Arch from *d* to *e*. Then from B, through the Point *e*, draw a right Line, till it meet with the Perpendicular before drawn, which it will do in the Point C. And thus have you protracted your Triangle C A B. Lastly, take in your Compasses the length of the Hypotenuse C B, and measure it upon your Scale of equal parts, and you shall finde it to contain 225.

The Analogie or Proportion is,

As the Sine of C is to the Logarithm of A B,
So is the Radius to the Logarithm of C B.

Case VII.

The Base A B 180, *and the Perpendicular C A* 135, *being given, to finde the Hypotenuse C A.*

DRaw a right Line A B containing 180 equal parts, and upon the end A erect the Perpendicular A C, and out of your Scale of equal parts take the length thereof 135, which set from A to C, and draw the Line C B, which con-

ſtitutes the Triangle C A B. Laſtly, take the length of the Hypotenuſe C B in your Compaſſes, and meaſuring it upon your Line of equal parts, you ſhall finde it to contain 225.

The Analogie or Proportion is,

1. Operation.

As the Logarithm of A B is to the Logarithm of C A,
So is the Radius to the Tangent of B.

2. Operation.

As the Sine of B is to the Logarithm of C A,
So is the Radius to the Logarithm of C B.

Theſe are the ſeverall Varieties or Caſes that can at any time fall out in the Solution of Right-angled plain Triangles, wherefore we will now proceed to the Solution of Oblique plain Triangles.

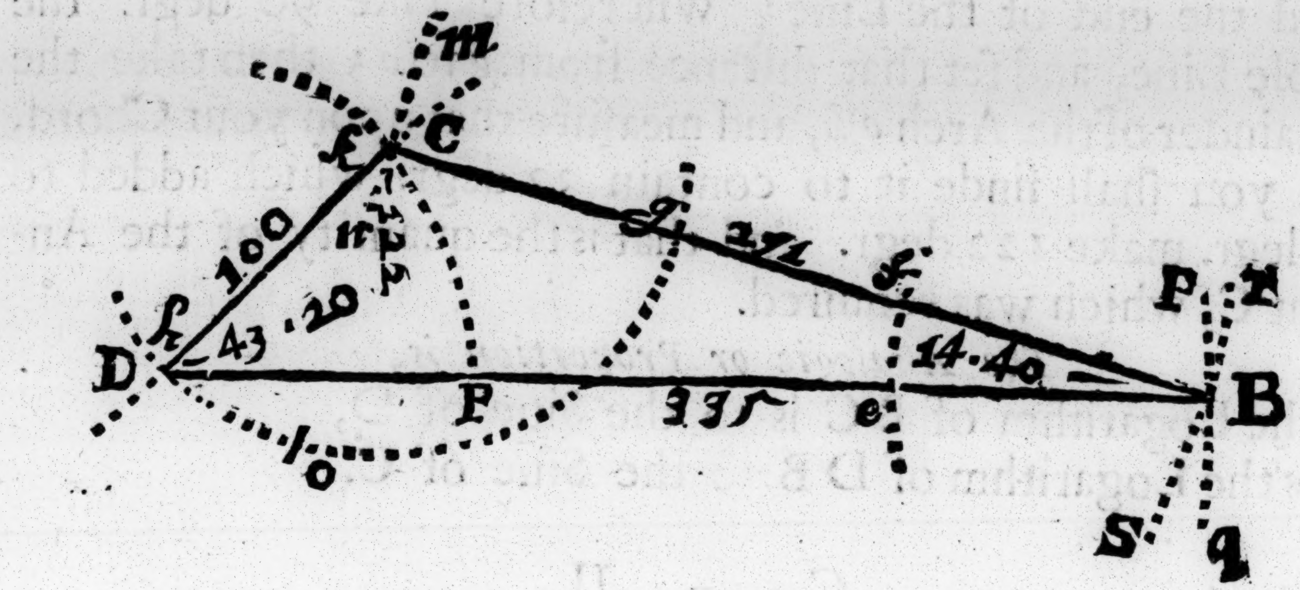
II. *Of Oblique-angled plain Triangles.*

THE *Triangle* which I ſhall make uſe of in the Solution of the ſeverall *Caſes* appertaining to an *Oblique-angled plain Triangle* ſhall be this following, C D B, in which

		parts
D B, the Baſe,	contains	335
C B, the longer Side,		271
D C, the ſhorter Side,		100

And		deg. m.
C, the obtuſe Angle,	contain	122—00
D, the 2 acute Angles,		43—20
B,		14—40

CASE



Case I.

Two Sides, as the Base DB 335, *and the Side C B* 271, *and the Angle D* 43 *degr.* 20 *min. opposite to C B, to finde the Angle at C, opposite to the Base D B.*

DRaw a right Line D B repreſenting the Baſe of your Triangle, which, by help of your Scale of equal parts, make to contain 335. Then upon the Point D, with the diſtance of 60 degr. of your Line of Chords, deſcribe the Arch *k l*, and from your Chords take 43 degr. 20 min. the quantity of the Angle at D, and ſet it upon the Arch-line from *l* to *k*, drawing the Line C D. And becauſe your other given Side B C contains 271 parts, take 271 out of your Line of equal parts, and ſetting one foot in B, with the other deſcribe the Arch *m n*, croſſing the former Arch *k l* in the Point C: then draw the Line C B. So ſhall you have conſtituted the Triangle C D B. Laſtly, becauſe it is the Angle at C that is required, take 60 degr. of your Chords, and upon G deſcribe the Arch *g h*, and taking the diſtance between *g* and *h*, apply it to your Line of Chords, and

and you ſhall finde it to reach from the beginning thereof beyond the end of the Line; wherefore take 90 degr. the whole Line, and ſet that diſtance from *g* to *o*; then take the remainder of the Arch *o h*, and meaſure that upon your Chord, and you ſhall finde it to contain 32 degr. which added to 90 degr. make 122 degr. and that is the quantity of the Angle at C, which was required.

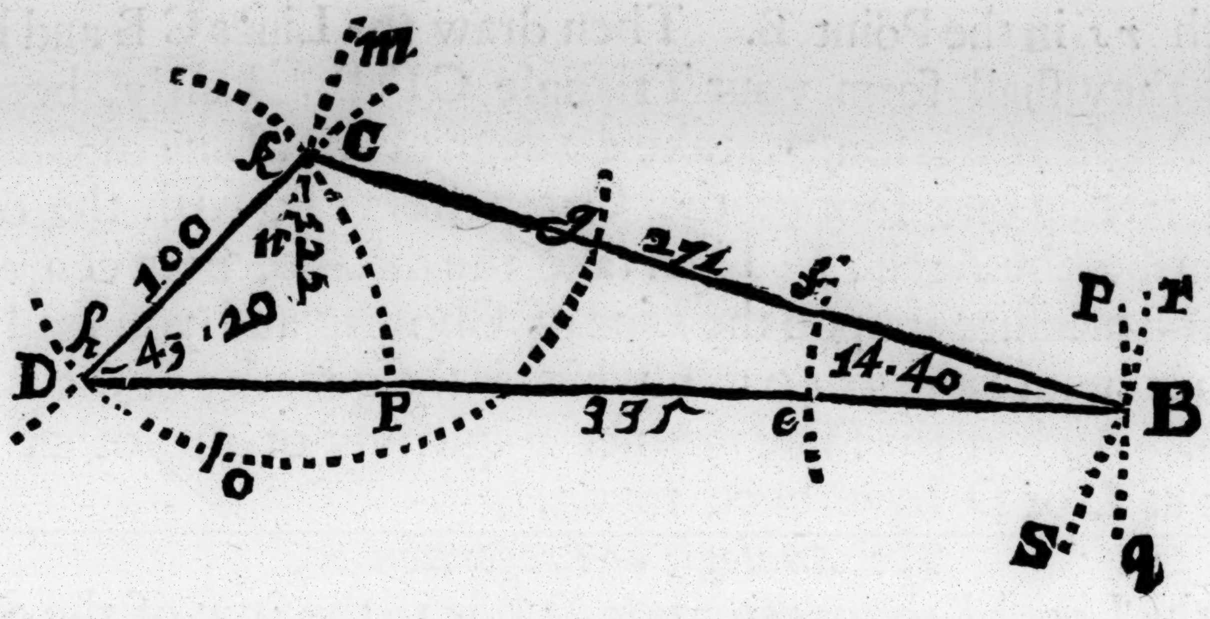
The Analogie or Proportion is,

As the Logarithm of B C is to the Sine of D,
So is the Logarithm of D B to the Sine of C.

Case II.

The Baſe D B 335, *and the Side D C* 100, *with the Angle D,* 43 *degr.* 20 *min. contained between them, to finde either of the other Angles at B and C.*

DRaw a right Line, as D B, containing 335 of your Scale of equal parts, which ſhall be the Baſe of your Triangle. Then with 60 degr. of your Line of Chords, upon the Point D deſcribe the Arch *k l*; and becauſe the given Angle at D contains 43 degr. 20 min. take 43 degr. 20 min. from your Line of Chords, and ſet it from *l* to *k*, drawing the Line D *k*. Again, becauſe the given Side D C contains 100, ſet 100 of your Line of equal parts from D to C; then drawing a right Line from C to B, you ſhall by that means find the oblique-angled Triangle C D B. Laſtly, being the other two Angles at B and C are to be found, with 60 degr. of your Chord on the Point B deſcribe the Arch *e f*; alſo upon the Point C deſcribe the Arch *g o h*. Then if you take the diſtance between *e* and *f* in your Compaſſes, and meaſure it upon your Line of Chords, you ſhall finde it to contain 14 degr. 40 min. And that is the quantity of the Angle at B. Then being the Angle at C, which is alſo required, is obtuſe, and contains above 90 degr. take 90 degr. out of your Line of Chords, and



and set that distance upon the Arch *g h*, from *g* to *o*: and taking the other part of the Arch *o h* in your Compasses, measure that upon your Chord, and you shall find it to contain 32 degr. which added to 90 degr. makes in all 122 degr. And such is the quantity of the other enquired Angle at C.

The Analogie or Proportion is,

As the Log. of the Sum of the two Sides given, C D and C B, is to the difference of those Sides,

So is the Tang. of half the Sum of the two unknown Angles, C and B, to the Tangent of half their difference.

Case III.

The three Sides, DB 335, *CB* 271, *and DC* 100, *being given, to finde any of the Angles, as B.*

DRaw a right Line C D, containing 100 of your Line of equal parts. Then the other side of the Triangle being 271, take 271 out of your Scale of equal parts, and setting one foot of the Compasses in C, with the other describe the Arch *r s*. Also take the length of your Base 335 out of your Line of equal parts, and setting one foot of the Compasses in

D, with the other describe the Arch *p q*, crossing the former Arch *r s* in the Point B. Then draw the Lines C B and D B, and they shall form your Triangle C D B. Lastly, because the Angle at B is sought, take 60 degr. of your Line of Chords, and setting one foot of the Compasses in B, with the other describe the Arch *e f*. Then take the distance between *e* and *f*, and measure it upon the Line of Chords, and you shall find it to contain 14 degr. 40 min. which is the quantity of the Angle at B. And in the same manner might any of the other Angles at C or D have been found.

The Analogie or Proportion is,

As the Log. of the greater side D B is to the Sum of the other two Sides, D C and C B,

So is the difference of the two Sides to a fourth Sum.

Which fourth Sum being taken from the Base, will leave another number, the half whereof will be the place in the Base where a Perpendicular let fall from the obtuse Angle would fall upon the Base: and so the oblique Triangle is reduced into two right-angled, and may be resolved by the Precepts of right-angled Triangles.

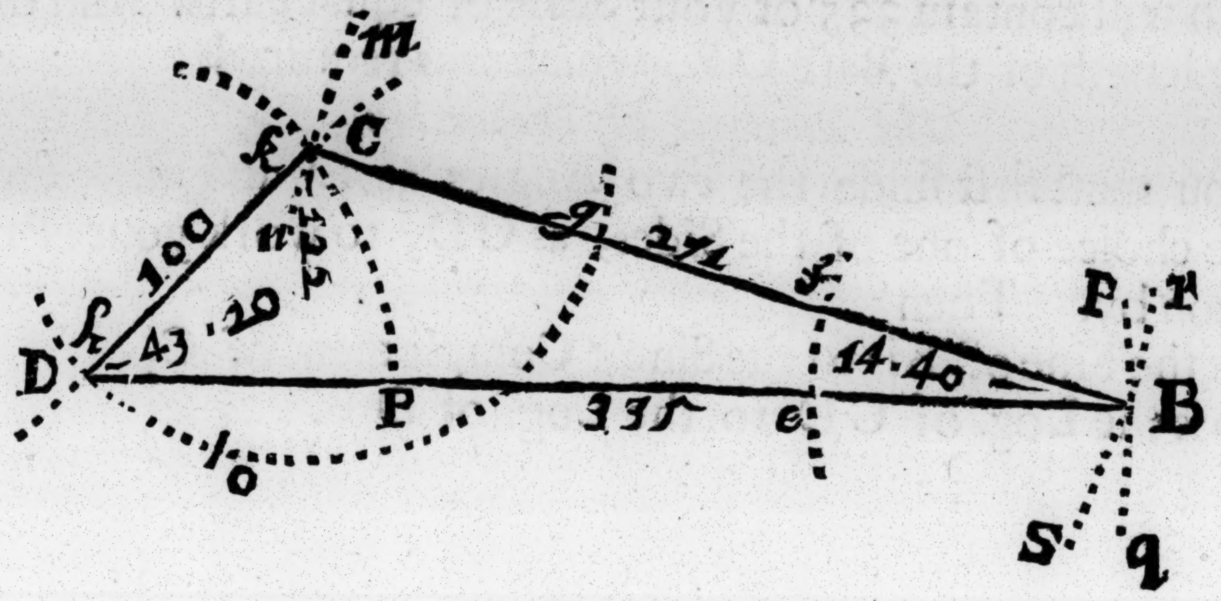
Case IV.

The three Angles, C 122 degr. D 43 degr. 20 min. and B 14 deg. 40 min. being given, to finde any of the Sides, as B C.

IN this Case, where the three Angles are given, and none of the Sides, you are to take notice, that the Sides cannot be absolutely found themselves, but the Proportionality of them. Wherefore

Draw a Line, as D B, of any length, and taking 60 degr. of your Line of Chords, set one foot of the Compasses upon D, and with the other describe the Arch *l k*; also set one foot in B, and with the other describe the Arch *e f*. Then, because the Angle at D contains 43 degr. 20 min. take 43 degr.

20 min.



20 min. from your Line of Chords, and set them from *l* to *k*. Also the Angle at B being 14 degr. 40 min. take them likewise from your Line of Chords, and set them from *e* to *f*. This done, draw the Lines B *f* and D *k*, extending them till they meet one with another, which they will doe in the Point C. So have you constituted the Triangle C D B, the Sides whereof will be in proportion the one to the other as the Sides of this Triangle are.

Case V.

The two Sides D C 100, *and C B* 271, *with the Angle at C* 122 *degr. being given, to finde the Base D B.*

DRaw a right Line, as C B, containing 271 of your Line of equal parts, and on the Point C, with 60 degr. of your Line of Chords, describe the Arch *g o h*. Then, because the given Angle C contains 122 degr. it being 32 degr. above 90 degr. first take 90 degr. from off your Line of Chords, and set it upon the Arch from *g* to *o*; and then take 32 degr. from your Chord also, and set them upon the same Arch from *o* to *h*, and draw the Line C *h*: then take 100, the length of the other given Side, and set it from C to D, and draw the Line D B,

which will contain 335 of your Scale of equal parts. And that is the length of the Base D B, which was required.

The Analogie or Proportion.

You must first finde the two Angles at D and B. Then make choice of one of the Sides, as C D, to work your Proportion by. Then

As the Sine of B is to the Sine's Complement of C,
So is the Log. of C D to the Log. of D B.

The End of the Doctrine of the Dimension of right-lined Triangles.

THE

THE DIMENSION Of Sphericall TRIANGLES BY A LINE OF CHORDS.

The Third EXERCISE.

As *the* Sides *of* Right-lined Triangles *were measured by* Equal parts *and their* Angles *by a* Line of Chords; *so the* Sides *of* Sphericall Triangles *being Arches of* great Circles *of the* Sphere, *both* they *and the* Angles *also may be measured by Chords onely.*

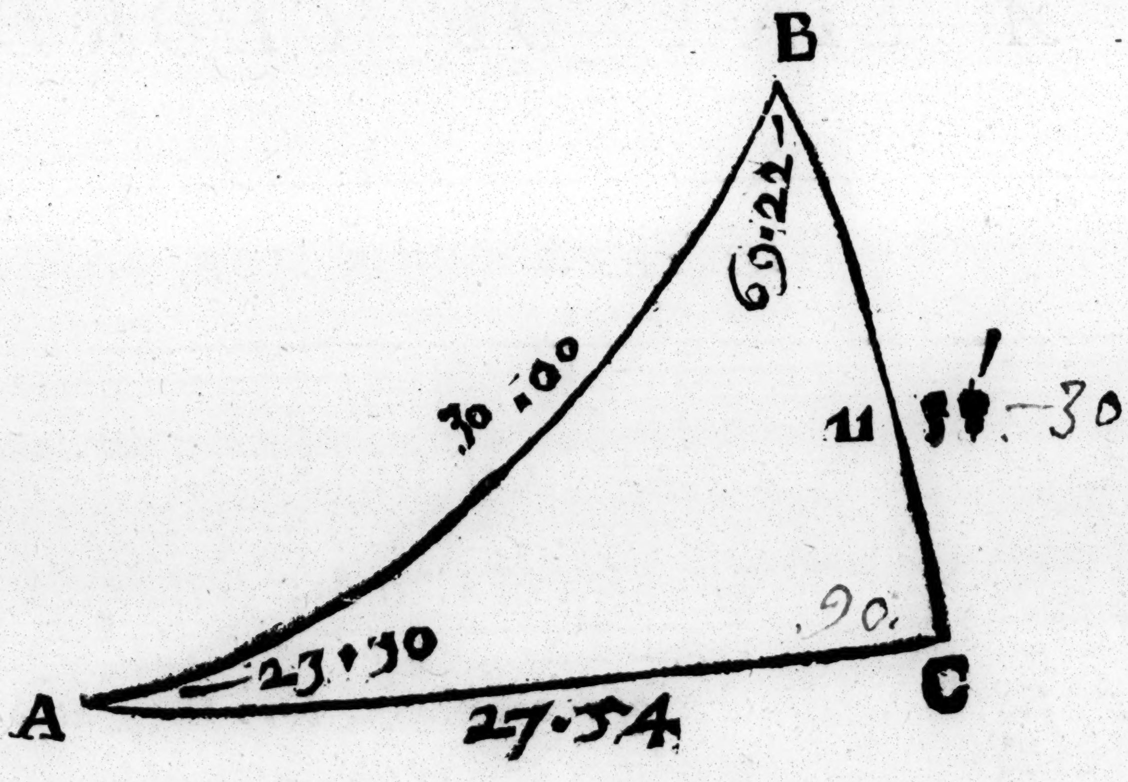
Of Sphericall Triangles *there be two kinds, as there were of* Right-lined; *namely*, right-angled *and* oblique-angled: *of both which there are* 28 Cases, *viz.* 16 *of* right-angled, *and* 12 *of* oblique-angled. *And to*

observe the same Method in these *as I did in* plain Triangles, *I will begin with the Solution*

I. *Of Right-angled Sphericall Triangles.*

THE *Right-angled sphericall Triangle* which I shall make use of for my *Examples* shall be this following, A B C, whose Sides and Angles contain as followeth.

			deg.	m.
The Side	A B	containeth	30	00
	B C		11	30
	A C		27	54
The Angle at	A	containeth	23	30
	B		69	22
	C		90	00



CASE

CASE I.

The Base A C 27 degr. 54 min. and the Perpendicular C B 11 degr. 30 min. being given, to finde the Hypotenuse A B.

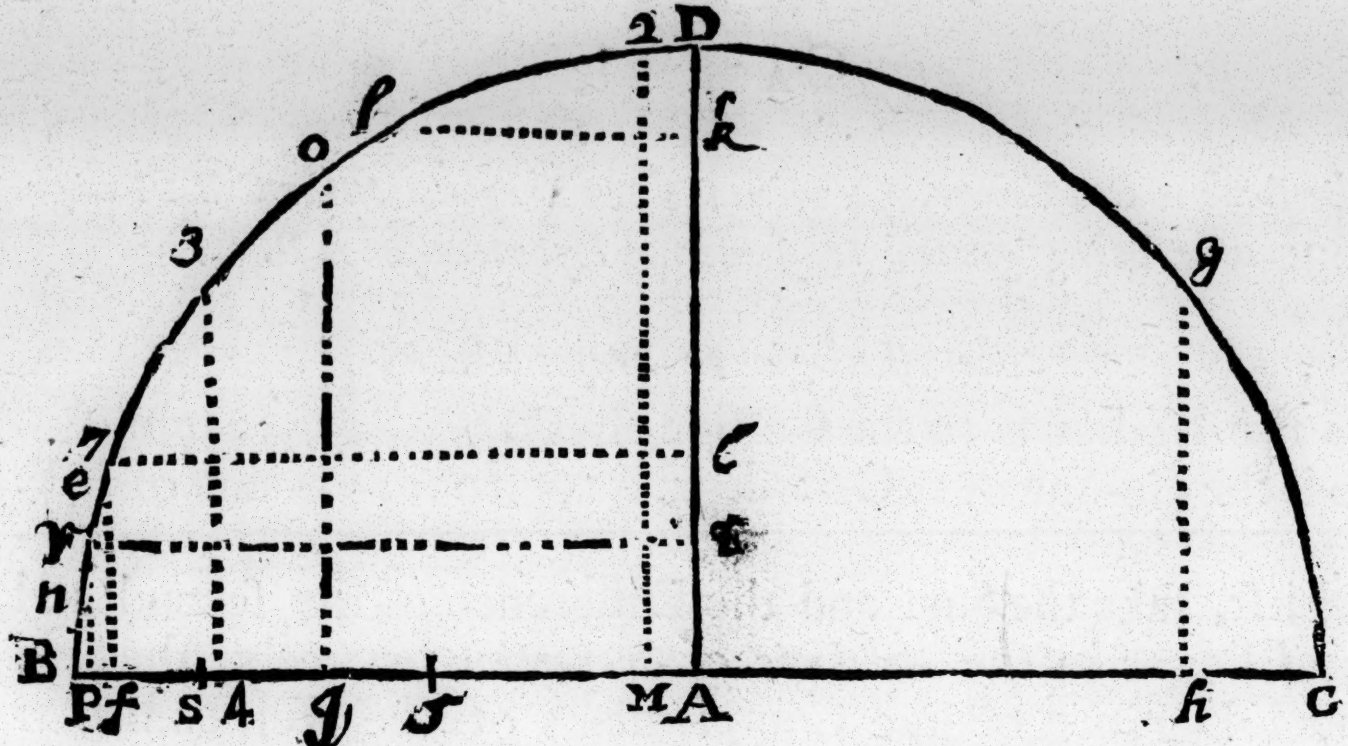
The Analogie or Proportion is,

As the Radius is to the Co-sine of B C 78 degr. 30 min.
So is the Co-sine of A C 62 degr. 6 min. to the Co-sine of A B.

First, take the Sum and the Difference of the second and third Terms in the *Analogie* or *Proportion*; namely, the Sum and Difference of the Complements of the Perpendicular B C 11 degr. 30 min. and the Base A C 27 degr. 34 min.

	degr.	m.			
The Base A C is	27	54.	its Comp.	62	06
The Perpendicular B C is	11	30.	its Comp.	78	30
			Sum	140	36
			Differ.	16	24

HAving found this Sum and Difference, draw a right Line B A C; then take 60 degr. out of your Line of Chords, and setting one foot of the Compasses in A, with the other describe the Semicircle B D C, and upon the Centre A erect the Perpendicular A D. This done, take 16 degr. 24 min. (the Difference) out of your Line of Chords, and set them from B to *e*; also take the Sum 140 degr. 36 min. (or rather 39 degr. 24 min. the Complement thereof to 180 degr.) and set them from C to *g*, and from the Points *e* and *g* let fall the two Perpendiculars *e f* and *g h*: then divide the space between *f* and *h* into two equal parts in M, and set the distance



M *h* or M *f* from A to *k*, and draw the Line *k l* parallel to A B, cutting the Semicircle in *l* : so shall D *l* be the quantity of the Hypotenuse A B, which if you measure upon your Line of Chords, you will finde to contain 30 degr.

CASE II.

The Hypotenuse A B 30 *degr. and the Angle at the Base A* 23 *degr.* 30 *min. being given, to find the Perpendicular B C.*

The Analogie or Proportion is,

As the Radius is to the Sine of the Hypotenuse A B 30 degr.
So is the Sine of the Angle at A 23 degr. 30 min. to the Sine of the Perpendicular B C, 11 degr. 30 min.

Take the Sum and Difference of the Angle at A 23 degr. 30 min. and the Hypotenuse A B 30 degr.

	degr.	m.
The Sum is	53	30
The Difference is	6	30

Having

HAving drawn your Semicircle B D C, as in the laſt Caſe, take 6 degr. 30 min. the Difference, out of your Line of Chords, and ſet them upon the Semicircle from B to *n*; alſo take 53 degr. 30 min. from your Line of Chords, and ſet them upon your Semicircle from B to *o*, and from the Points *n* and *o* let fall two Perpendiculars *n p* and *o q*. Then divide the ſpace between *p* and *q* into two equal parts in *s*; then take the diſtance *q s*, and ſet it from A to *t*, and through the Point *t* draw a Line *t r* parallel to A B: ſo ſhall B *r*, being meaſured upon your Line of Chords, contain 11 degr. 30 min. which is the quantity of the Perpendicular B C.

CASE III.

The Baſe A C 27 *degr.* 54 *min. and the Angle at the Baſe A* 23 *d.* 30 *min. being given, to find the Angle at the Perpendicular B.*

The Analogie or Proportion is,

As the Radius is to the Co-ſine of A C 62 degr. 6 min.
So is the Sine of the Angle at A 23 degr. 30 min. to the Co-ſine of the Angle at B.

Take the Sum and Difference of the Co-ſine of A C 62 d. 6 min. and of the Sine of the Angle at A 23 degr. 30 min.

	degr.	m.
The Sum is	85	36
The Difference is	38	36

BEing thus far prepared, and having drawn your Semicircle B D C, out of your Line of Chords take 85 degr. 36 m. and ſet them upon your Semicircle from B to 2; alſo take the Difference 38 degr. 36 min. from your Chord, and ſet them from B to 3, and let fall the two Perpendiculars, 3 4, and 2 M. Then divide the diſtance between M and 4 into two

equal parts in the Point 5, and taking the diſtance M 5 in your Compaſſes, ſet it upon the Line A D, from A to 6. Laſtly, draw the Line 6 7 parallel to B A; ſo ſhall D 7 being meaſured upon your Line of Chords contain 69 degr. 22 min. the quantity of the Angle at B required.

Theſe three Caſes are all (in Sines alone) that have the Radius in the firſt place of the Analogie or Proportion, and ſo conſequently all that can be wrought by this Artifice: wherefore thoſe which follow muſt be reſolved by other means.

Case IV.

The Perpendicular B C 11 *degr.* 30 *min. and the Angle at the Baſe A* 23 *degr.* 30 *min. to finde the Hypotenuſe A B.*

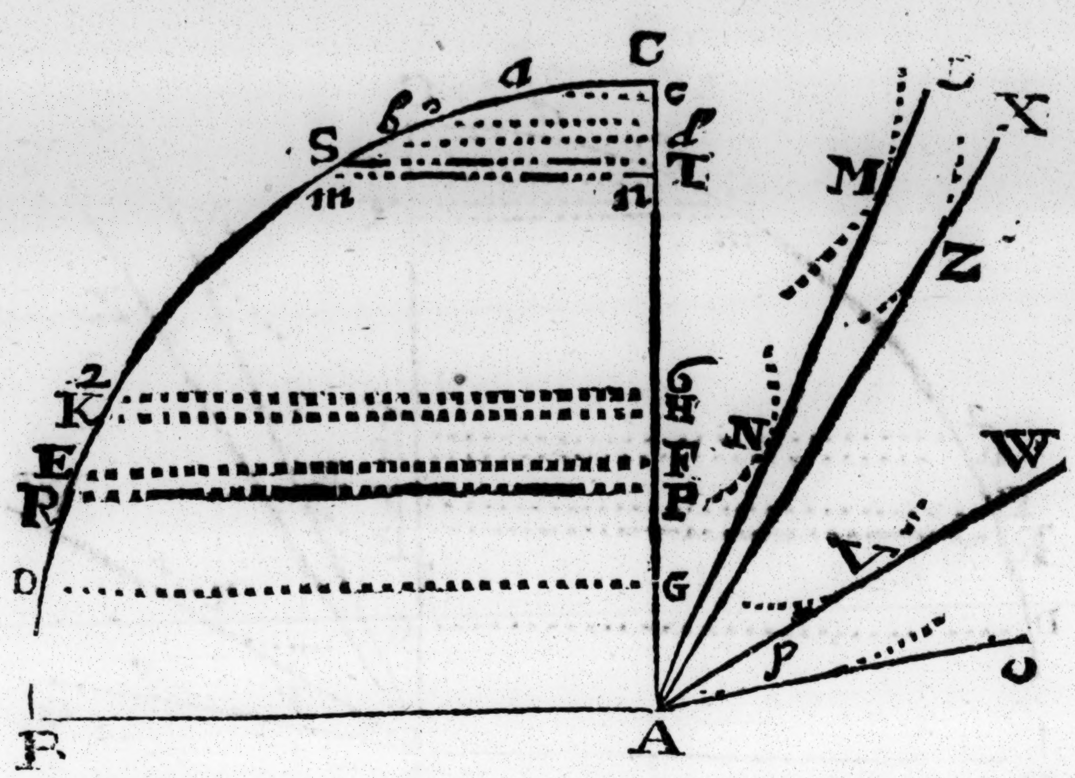
The Analogie or Proportion is,

As the Sine of the Angle at A 23 degr. 30 min. is to the Sine of the Side B C 11 degr. 30 min.
So is the Radius to the Sine of A B.

First, draw a right Line B A, and upon the Point A, with 60 degr. of your Line of Chords, deſcribe the Quadrant A B C.

Then take 23 degr. 30 min. the quantity of the given Angle at A, out of your Line of Chords, and ſet them upon the Quadrant from B to E. Alſo take 11 degr. 30 min. the quantity of the given Side B C, out of your Line of Chords, and ſet them from B to D; and draw the Lines E F and D G both parallel to B A.

Then with your Compaſſes take the diſtance between A and F, and ſetting one foot in C, with the other deſcribe the Arch M, and from A draw the Line A L ſo that it may onely touch the Arch M. Then taking the diſtance A G in your Compaſſes, ſet one foot of them upon the Line A C, and draw it gently



gently and softly along the Line A C, till the other foot, being turned about, will onely just touch the Line A L; and when it so toucheth, mark where the other Point resteth upon the Line A C, which you shall finde it to doe at the Point 6. Lastly, through the Point 6 draw the Line 6 2, parallel to A B. So shall the distance 2 B 6, being measured upon the Line of Chords, contain 30 degr. the quantity of the Hypotenuse A B, which was the Side required.

Case V.

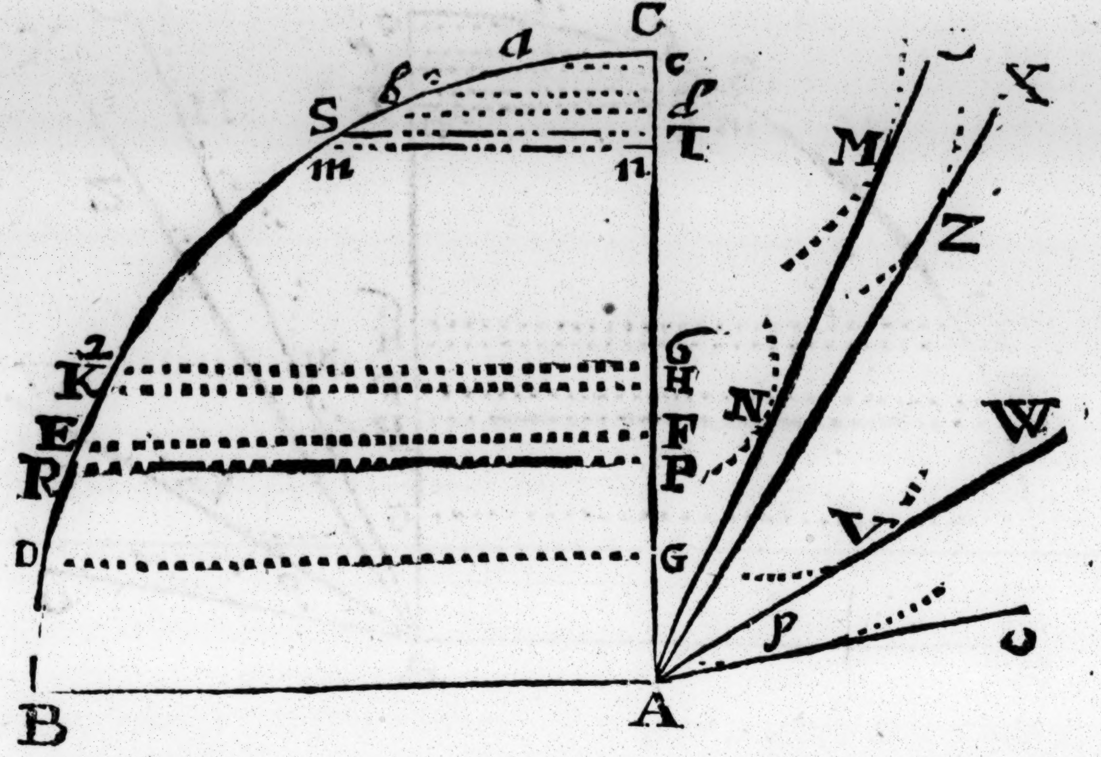
The Perpendicular B C 11 *degr.* 30 *min. and the Angle at the Base A* 23 *degr.* 30 *min. being given, to finde the Angle at the Perpendicular B.*

The Analogie or Proportion is,

As the Co-sine of B C 78 degr. 30 min. is to the Co-sine of A 66 degr. 30 min.

So is the Radius to the Sine of the Angle at the Perpendicular B.

 Having



HAving drawn your Quadrant A B C, take from your Line of Chords 78 degr. 30 min. the Complement of the given Side B C, and ſet it from B to *a*. Alſo take from your Chords 66 degr. 30 min. the Complement of the given Angle at A, and ſet them from B to *b*, and through the Points *a* and *b* draw the Lines *a c* and *b d* parallel to B A. Then taking in your Compaſſes the diſtance A *c*, ſet one foot in C, and with the other foot deſcribe the Arch *p*, and draw the Line A O ſo that it onely touch the Arch *p*. Likewiſe, take in your Compaſſes the diſtance A *d*, and with that diſtance, ſetting one foot upon the Line A C, move it gently along that Line till the other foot, being turned about, do onely touch the Line A O. So ſhall you finde the point of the Compaſſes to reſt in the Point *e*, through which Point draw the Line *e o* parallel to B A. Then meaſure the diſtance B *o* upon your Line of Chords, and you ſhall finde it to contain 69 degr. 22 min. the quantity of the enquired Angle at B.

CASE

CASE VI.

The Angle at the Base A 23 degr. 30 min. and the Angle at the Perpendicular B 69 degr. 22 min. being given, to finde the Base A C.

The Analogie or Proportion is,

As the Sine of the Angle at A 23 degr. 30 min. is to the Co-sine of the Angle at B 20 degr. 38 min.
So is the Radius to the Co-sine of the Base A C.

YOur Quadrant being drawn, take 23 degr. 30 min. the quantity of the Angle at A, and set it from B to E. Also take 20 degr. 38 min. the Complement of the Angle at B, out of your Line of Chords, and set them upon your Quadrant from B to R, and through the Points E and R draw the Lines E F and R P parallel to A B.

Then take in your Compasses the distance A F, and setting one foot in C, with the other describe the Arch M, and draw the Line A L so that it onely touch the Arch M. Then in your Compasses take the distance A P, and setting one foot upon the Line A C, move it along till the other, being turned about, do onely touch the Line A L; and when it so toucheth, note upon what part of the Line A C the Compass-point resteth, which you will finde it to doe at the Point T; through that Point draw the Line T S parallel to B A. So shall the distance B S, being measured upon your Line of Chords, contain 62 d. 6 min. the Complement of the enquired Side: or, S C will give 27 degr. 54 min. the Side it self.

Case VII.

The Base A C 27 degr. 54 min. and the Hypotenuse A B 30 degr. being given, to finde the Angle at the Perpendicular B.

The Analogie or Proportion is,

As the Sine of the Hypotenuse A B 30 degr. is to the Radius,
So is the Sine of the Base A C 27 degr. 54 min. to the Sine of the Angle at the Perpendicular B.

Having drawn your Quadrant, take from your Line of Chords 30 degr. the quantity of the Hypotenuse, and set them upon your Quadrant from B to 2; also take 27 degr. 54 min. the quantity of the Base, from your Chords, and set them from B to K, and draw the Lines 2 6 and K H, both parallel to the Line B A. Then taking in your Compasses the distance A 6, set one foot of them in C, and with the other describe the Arch Z, and draw the Line A X, so that it onely touch the Arch Z. Then taking in your Compasses the distance A H, set one foot upon the Line A C, moving it along till the other foot, being turned about, will onely touch the Line A Z; and where the point of the Compasses resteth upon the Line A C, which it will doe at *e*, through that Point draw the Line *o e* parallel to B A. So shall B *o*, being measured upon your Line of Chords, give you 69 degr. 22 min. the quantity of the enquired Angle at B.

Case VIII.

The Base A C 27 degr. 54 min. and the Hypotenuse A B 30 degr. being given, to finde the Perpendicular B C.

The Analogie or Proportion is,

As the Co-sine of the Base A C 62 degr. 6 min. is to the Radius,
So is the Co-sine of the Hypotenuse 60 degr. to the Co-sine of the Perpendicular B C.

Having

Having drawn your Quadrant A B C, take out of your Line of Chords 62 degr. 6 min. the Co-ſine of the Baſe A C, and ſet them from B to S. Alſo take from the Chords 60 degr. the Co-ſine of the Hypotenuſe, and ſet them from B to *m*: and draw the Lines S T and *m n* both parallel to B A. Then taking the diſtance A T in your Compaſſes, ſet one foot in C, and with the other deſcribe the Arch V, and draw the Line A W ſo that it may onely touch the Arch V. Then taking A *n* in your Compaſſes, move one foot thereof gently along the Line C A, till the other, being turned about, doth onely touch the Line A W; and where the Point reſteth upon the Line C A, which you will finde to be at *c*, there make a mark, and draw the Line *c a* parallel to B A. Laſtly, take the diſtance from B to *a*, and meaſure it upon your Line of Chords, where you ſhall finde it to contain 78 degr. 30 min. the Complement of the Perpendicular; or, C A meaſured upon the Chord will give you 11 d. 30 min. the Perpendicular it ſelf.

Theſe Five laſt are all the Caſes in a Right-angled Sphericall Triangle that are reſolvable by Sines alone. Thoſe which follow are to be reſolved by Sines and Tangents joyntly, and ſo will require another manner of Operation then the former.

Case IX.

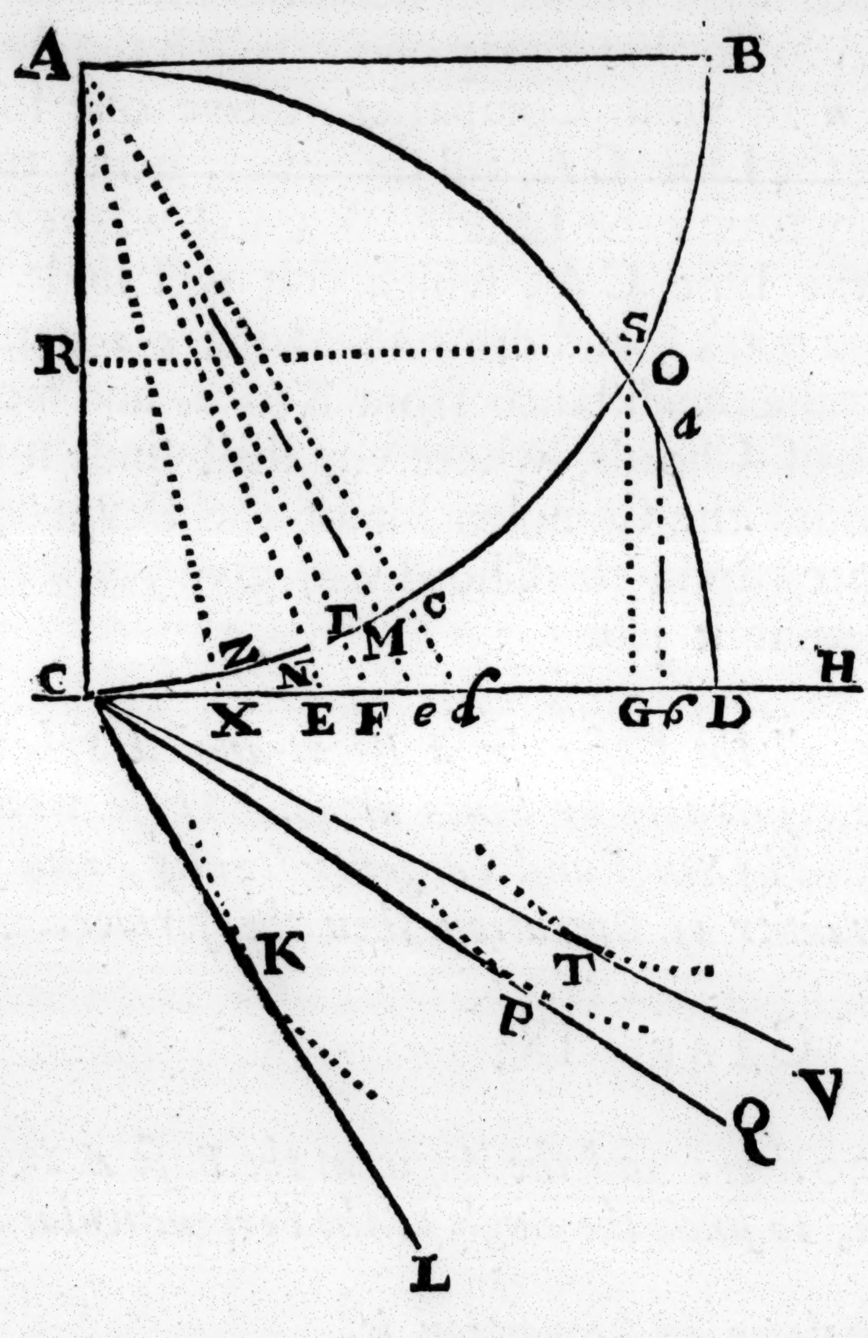
The Hypotenuſe A B 30 degr. and the Angle at the Baſe A 23 d. 30 min. being given, to finde the Angle at the Perpendicular B.

The Analogie or Proportion is,

As the Radius is to the Co-ſine of the Hypotenuſe A B 60 deg.
So is the Tangent of the Angle at the Baſe A 23 degr. 30 min.
to the Co-tangent of the Angle at the Perpendicular B.

First,

First, draw a right Line, as C H, and upon one end thereof, (as at C) erect the Perpendicular C A, and with the distance of 60 degr. of your Line of Chords, upon the Centre C describe the Quadrant C A D. Also upon the Point A with 60 degr. of your Chord describe the Quadrant A B C.



Being thus prepared, First, take 60 deg. the Co-sine of the Hypotenuse A B, and set them from A to O; also take 23 d. 30 m. the quantity of the Angle at A, and set them from C to r; and draw the Line A M F, and the Line O G, parallel to A C.

Then take in your Compasses the distance between C and G, and setting one foot in D, with the other describe the Arch K, and draw the Line A L so that it onely touch the Arch K. Then placing one foot of the Compasses in F, take the least distance you can to the Line C L, which set from C to E. Lastly, draw the Line A E, cutting the Quadrant C B in N. So the distance

C N

C N meaſured upon your Chords ſhall give you 20 degr. 38 min. the Complement of the Angle at B, which was required; or, the diſtance B N will give you 69 degr. 22 min. the Angle it ſelf.

Case X.

The Hypotenuſe A B 30 degr. and the Angle at the Baſe A 23 d. 30 min. being given, to finde the Baſe A C.

The Analogie or Proportion is,

As the Radius is to the Co-ſine of the Angle at A 66 d. 30 m.
So is the Tangent of A B the Hypotenuſe 30 degr. to the Tangent of the Baſe A C.

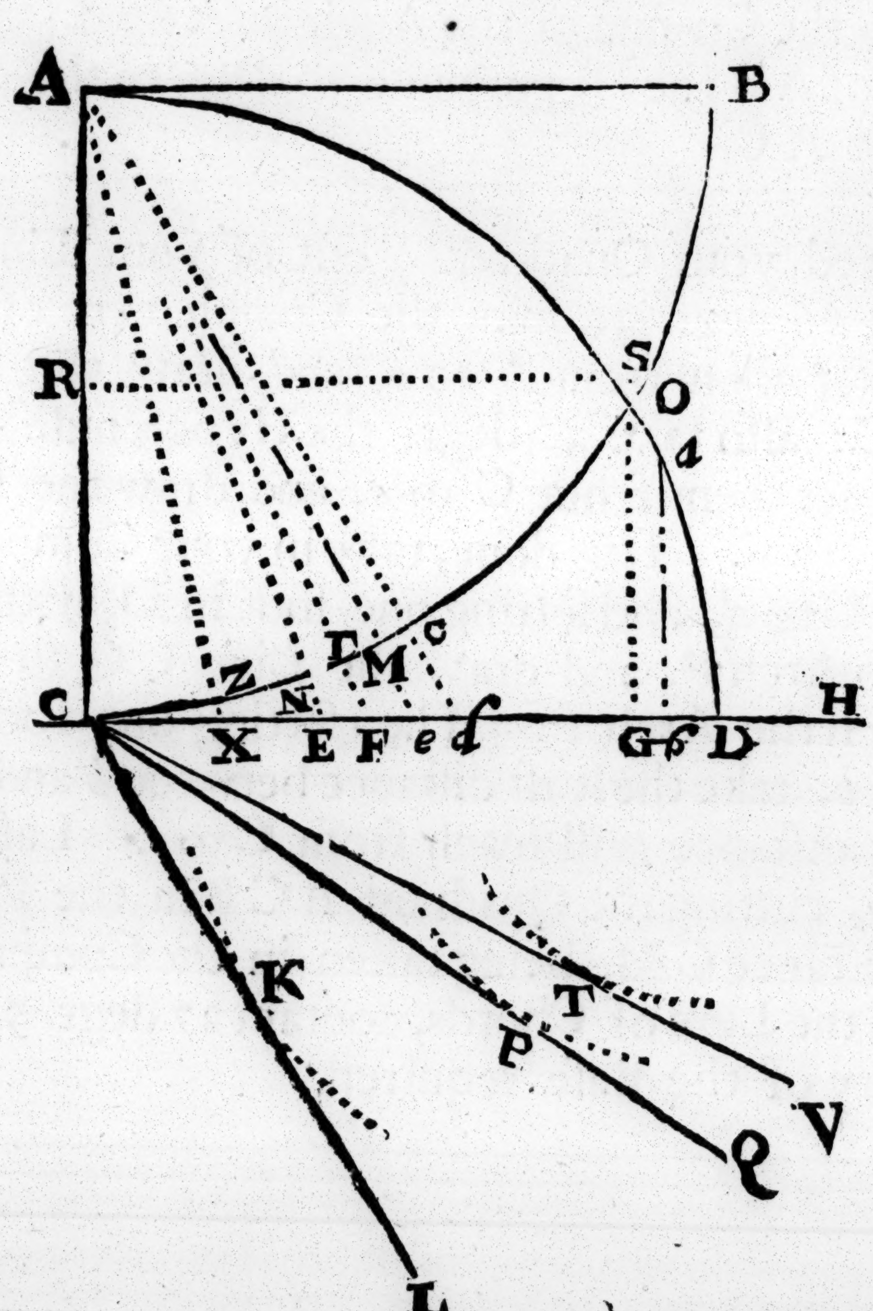
HAving prepared your Quadrants, out of your Line of Chords take 66 degr. 30 min. the Complement of the Angle at A, and ſet them from A to *a*, and draw the Line *a b* parallel to A C: alſo take 30 degr. the Hypotenuſe from your Chord, and ſet them from C to *c*, and draw the Line A *c*, prolonging it to *d*. This done, take in your Compaſſes the diſtance from C to *d*, and ſetting one foot in D, with the other deſcribe the Arch P, and draw the Line C Q ſo that it may onely touch the Arch P. Then ſetting one foot of the Compaſſes in *b*, take the leaſt diſtance between *b* and the Line C Q, which diſtance will reach from C to *e*. Laſtly, draw the Line A *e*, cutting the Quadrant A C B in the Point M. So ſhall the diſtance C M, being taken in the Compaſſes and meaſured upon the Line of Chords, contain 27 degr. 54 m. which is the quantity of the Baſe required.

CASE XI.

The Base A C 27 degr. 54 min. and the Angle at the Base A 23 degr. 30 min. being given, to finde the Perpendicular B C.

The Analogie or Proportion is,

As the Radius is to the Sine of the Base A C 27 degr. 54 min.
So is the Tangent of the Angle at A 23 degr. 30 min. to the Tangent of the Perpendicular B C.



YOur Quadrants being prepared, out of your Line of Chords take 27 d. 54 m. the quantity of the given Base A C, and set them from A to S, drawing the Line S R parallel to C D. Also take out of your Chords 23 degr. 30 minutes, the quantity of the given Angle at A, and set them from C to r, and draw the Line A r, prolonging it to F.

This done, take

take in your Compasses the distance A R, and setting one foot in D, with the other describe the Arch T, and by the side thereof draw the Line C V onely to touch it. Then set one foot of your Compasses in F, and with the other take the nearest distance to the Line C V, which distance will reach from C to X. Lastly, draw the Line A X, which will cut the Quadrant A B C in the Point Z. So shall C Z, being measured upon the Line of Chords, contain 11 degr. 30 min. which is the quantity of the Perpendicular B C, which was required.

Case XII.

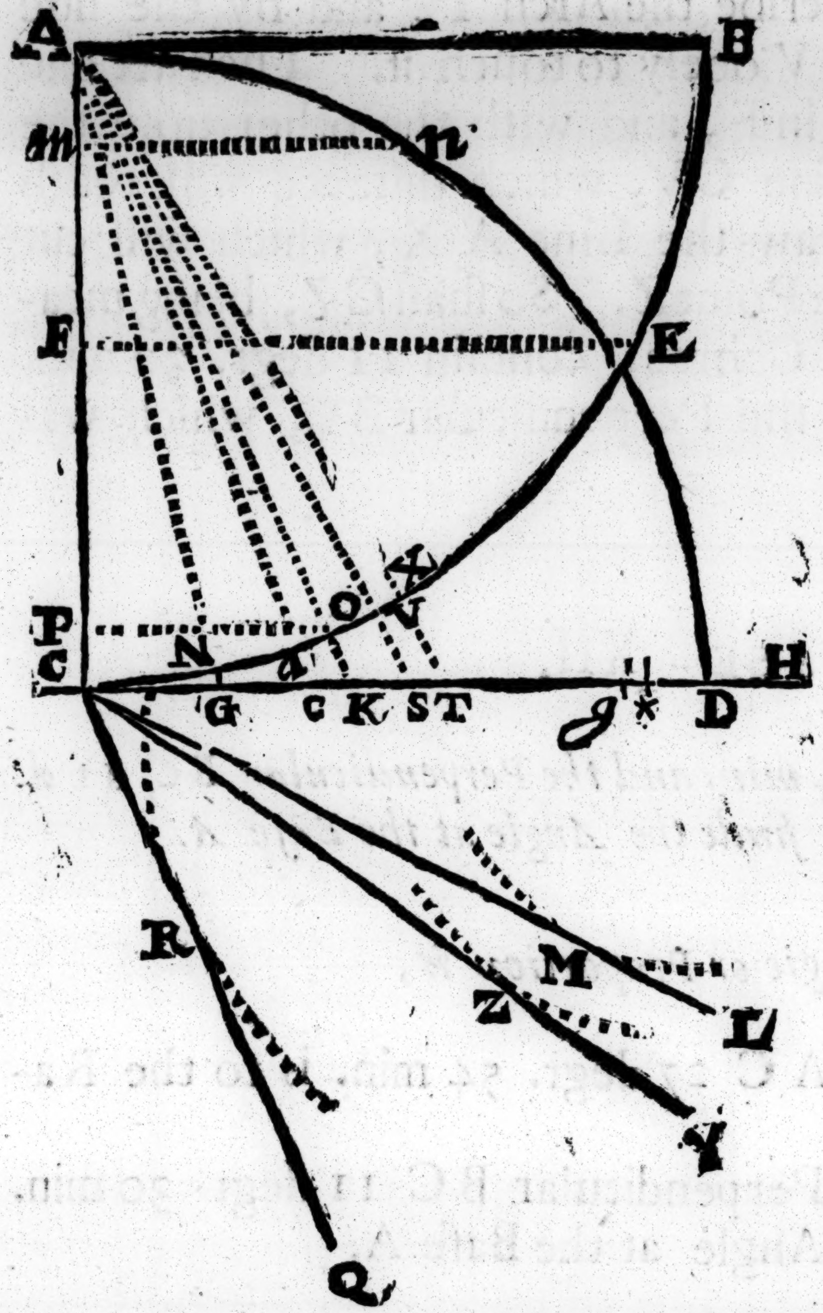
The Base A C 27 degr. 54 min. and the Perpendicular B C 11 d. 30 min. being given, to finde the Angle at the Base A.

The Analogie or Proportion is,

As the Sine of the Base A C 27 degr. 54 min. is to the Radius,
So is the Tangent of the Perpendicular B C 11 degr. 30 min. to the Tangent of the Angle at the Base A.

DRaw the two Quadrants A B C and C A D, as before. Then take out of your Line of Chords 27 degr. 54 m. the quantity of the given Base A C, and set them upon the Quadrant from B to E, and draw the Line E F parallel to C D. Also take 11 degr. 30 min. the quantity of the Perpendicular B C, and set them upon the Quadrant from C to N, and draw the Line A N, cutting the Line C H in G.

This done, take in your Compasses the distance A F, and setting one foot in D, with the other describe the Arch M, and close by the out-side of it draw the Line C L. Then



take in your Compasses the distance C G, and setting one foot of them upon the Line C D, move it along till the other foot, being turned about, will onely touch the Line C L; and when it so toucheth, mark where the other foot resteth upon the Line C D, which it will do at K. Lastly, draw the Line A K, cutting the Quadrant C B in the Point O. So shall C O, being measured on the Line of Chords, contain 23 degr. 30 min. the quantity of the enquired Angle at A.

Case XIII.

The Base A C 27 degr. 54 min. and the Angle at the Base A 23 d. 30 min. being given, to finde the Hypotenuse A B.

The Analogie or Proportion is,

As the Co-sine of the Angle at A 66 d. 30 m. is to the Radius,
So is the Tangent of the Base A C 27 degr. 54 min. to the Tangent of the Hypotenuse A B.

The

THE Quadrants being drawn, take the quantity of the given Baſe A C 27 degr. 54 min. out of the Line of Chords, and ſet them upon the Quadrant from C to V, and draw the Line A V, continuing it till it cut the Line C D in S. Alſo take 66 degr. 30 min. the Complement of the given Angle at A, out of the Chords, and ſet them from B to O, and draw the Line O P parallel to C D or A B.

This done, take the diſtance A P in your Compaſſes, and ſetting one foot in D, with the other deſcribe the Arch R, drawing the Line C Q onely to touch it. Then take in your Compaſſes the diſtance C S, and placing one foot of them upon the Line C D, move it along the ſame, till the other, being turned about, will onely touch the Line C Q; and where the Compaſs-point reſteth upon the Line C D, (which you will finde it to doe at T,) there make a mark, and from it draw the Line A T, cutting the Quadrant A B C in the Point X. So ſhall C X, being meaſured on the Line of Chords, give you 30 degr. the quantity of the Hypotenuſe required.

CASE XIV.

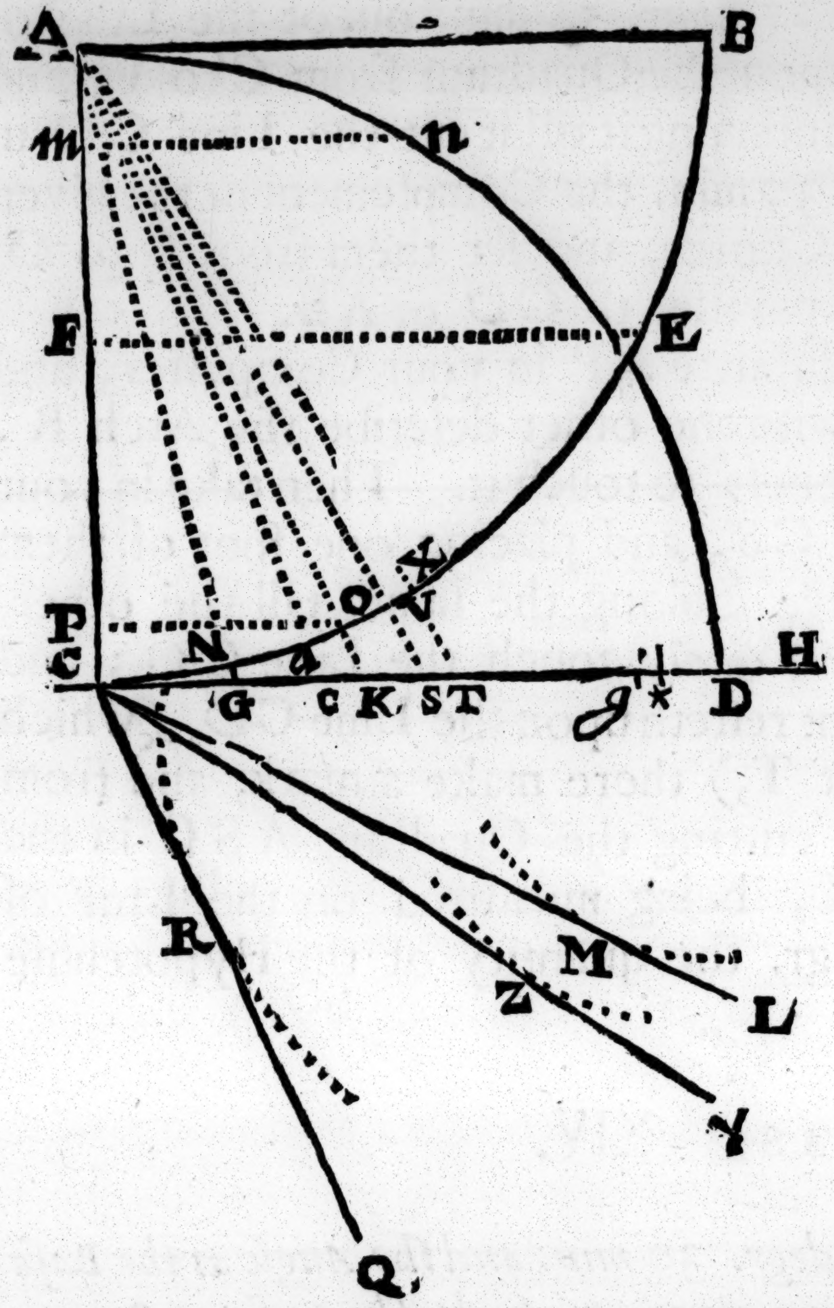
The Perpendicular B C 11 degr. 30 min. and the Angle at the Baſe A 23 degr. 30 min. being given, to finde the Baſe A C.

The Analogie or Proportion is,

As the Tangent of the Angle at A 23 degr. 30 min. is to the Tangent of the Perpendicular B C 11 degr. 30 min.
So is the Radius to the Sine of the Baſe A C.

HAving drawn your Quadrants, take out of your Line of Chords 23 degr. 30 min. the quantity of the Angle at A, and ſet them from C to O, and draw the Line A O, continuing it till it cut the Line C D in K. Alſo take out of your Line

of Chords 11 degr. 30 min. the quantity of the Perpendicular given, and set them from C to N, drawing the Line A N, and prolonging it to G.



This done, take in your Compasses the distance between C and K, and setting one foot in D, with the other describe the Arch M, and draw the Line C L. Then take in your Compasses the distance C G, and setting one foot upon the Line C D, move it gently along the same, till the other, being turned about, do only touch the Line C L; and when it so toucheth, keeping the Compasses at the same distance, set it from A to F. Lastly, draw the Line F E parallel to A B. So shall the distance B E, being measured on your Line of Chords, contain 27 degr. 54 min. the quantity of the Base A C, which was required.

CASE

CASE XV.

The Base A C 27 degr. 54 min. and the Hypotenuse A B 30 degr. being given, to finde the Angle at the Base A.

The Analogie or Proportion is,

As the Tangent of the Hypotenuſe A B 30 degr. is to the Tangent of the Baſe A C 27 degr. 54 min.
So is the Radius to the Co-ſine of the Angle at A.

THE Quadrants being drawn, out of your Line of Chords take 30 degr. the quantity of the Hypotenuſe given, and ſet them upon the Quadrant from C to X. Alſo take 27 degr. 54 min. the quantity of the given Baſe, from the Line of Chords, and ſet them upon the ſame Quadrant from C to V, and draw the Lines A X and A V, prolonging them to T and S.

This done, take in your Compaſſes the diſtance between C and T, and ſetting one foot in D, with the other deſcribe the Arch Z, and draw the Line C Y onely to touch it. Then take in your Compaſſes the diſtance C S, and placing one foot upon the Line C D, move it gently along, till the other, being turned about, do onely touch the Line C Y; and where the Point reſteth upon the Line C D make a mark, which will be at *. Then take the diſtance C *, and ſet it from A to P, and draw the Line P O parallel to C D. So ſhall B O, being meaſured upon the Line of Chords, give you 66 degr. 30 min. the Complement of the Angle at A, which was required; or, the diſtance O C upon the Chord ſhall give you 23 degr. 30 min. the Angle it ſelf.

CASE

Case XVI.

The Angle B at the Perpendicular 69 degr. 22 min. and the Angle at the Base A 23 degr. 30 min. being given, to finde the Hypotenuse A B.

The Analogie or Proportion is,

As the Co-tangent of the Angle at B 20 degr. 38 min. is to the Tangent of the Angle at A 23 degr. 30 min.
So is the Radius to the Co-sine of the Hypotenuse A B.

Having drawn the two Quadrants, as before, take out of your Line of Chords 23 degr. 30 min. the quantity of the Angle at A, and set them from C to O. Also take 20 degr. 38 min. from your Chord, (the Complement of the Angle at B) and set them from C to *a*, and draw the Lines A O and A *a*, continuing them to *c* and K upon the Line C D.

This done, take in your Compasses the distance C K, and setting one foot in D, with the other describe the Arch M, and draw the Line C L that it onely touch the Arch M. Also take in your Compasses the distance C *c*, and setting one foot upon the Line C D, move it gently along the same till the other, being turned about, do onely touch the Line C L; and where the Compass-point resteth upon the Line C D make a mark, as at *g*. Lastly, take the distance C *g*, and set it from C to *m*, and draw the Line *m n* parallel to A B. So shall the distance D *n*, measured on the Line of Chords, contain 60 d. the Complement of the Hypotenuse required; or, A *n* shall contain 30 degr. the Hypotenuse it self.

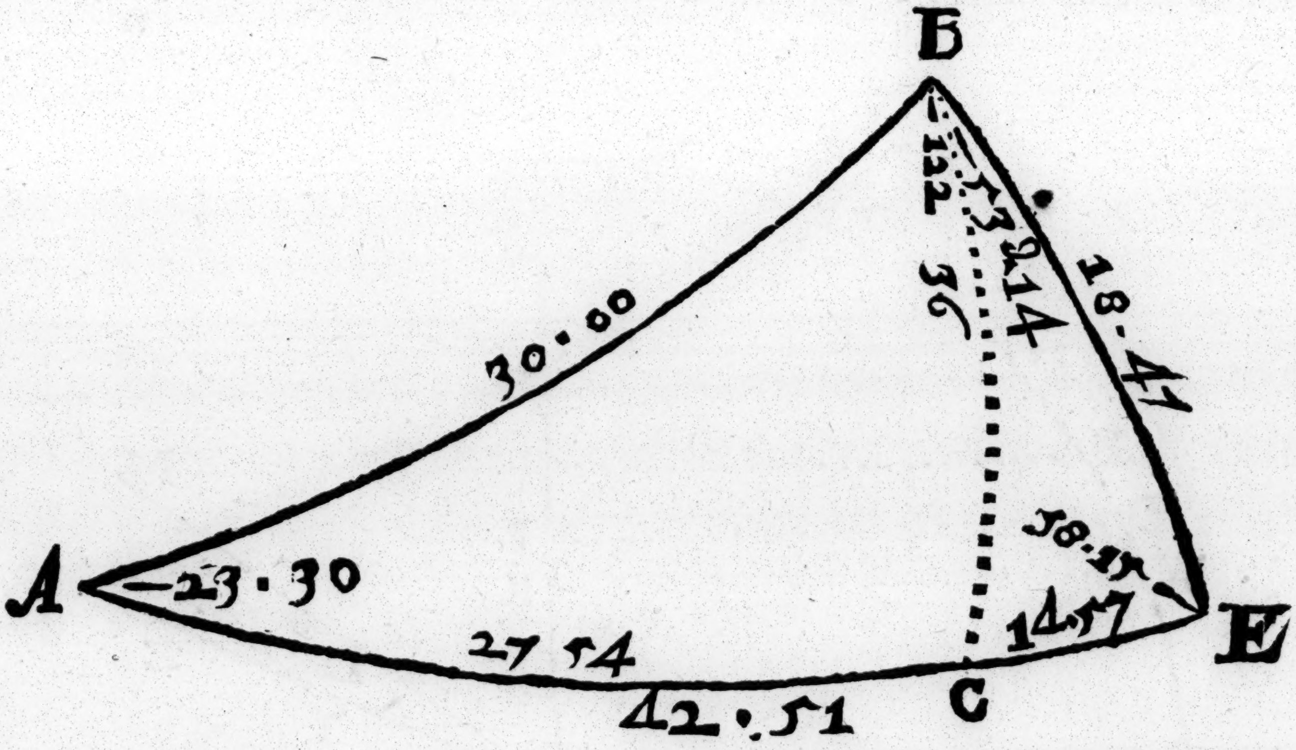
These 16 Cases are all that can be proposed in a Right-angled Sphericall Triangle. There are 12 other Cases which belong to Oblique-angled Sphericall Triangles, which we now come to resolve.

II. Of

II. *Of Oblique-angled Sphericall Triangles.*

THE *Triangle* that I shall make use of in the Solution of the 12 Cases of an *Oblique-angled Sphericall Triangle* shall be this, A B E: whose *Sides* and *Angles* are as followeth.

			degr.	m.
The Side	A B, the Base,	contains	30	00
	B E		18	47
	A E		42	51
The Angle	A	contains	23	30
	B		122	36
	E		38	15



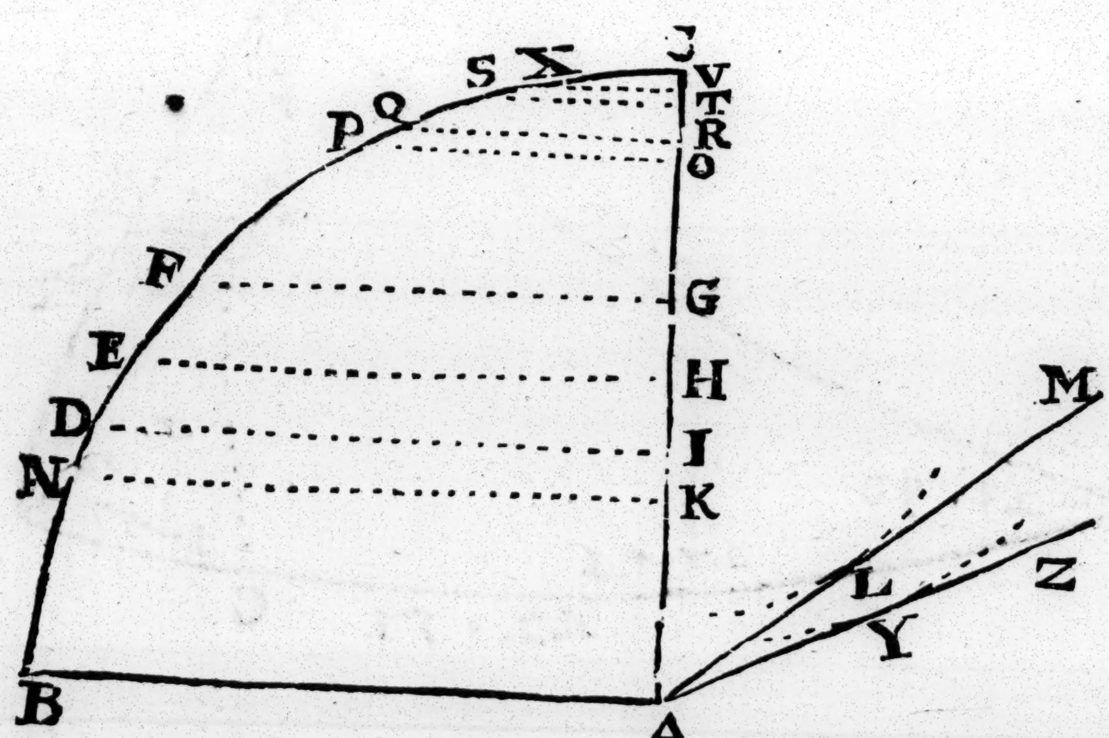
Case I.

The Angle at E 38 degr. 15 min. the Angle at A 23 degr. 30 m. and the Side A B 30 degr. being given, to finde the Base B E.

The Analogie or Proportion is,

As the Sine of the Angle at E 38 degr. 15 min. is to the Side A B 30 degr.
So is the Sine of the Angle at A 23 degr. 30 min. to the Sine of the Side B E.

First, draw the Quadrant A B C : then out of your Line of Chords take 38 degr. 15 min. the quantity of the Angle at E, and ſet it upon the Quadrant from B to F. Alſo take 30 degr. the quantity of the given Side A B, out of your Line of Chords, and ſet them upon the Quadrant from B to E, and draw the Lines F G and E H both of them parallel to A B.



This done, take in your Compaſſes the diſtance A H, and ſetting

ſetting one foot of that extent in G, with the other deſcribe the Arch L, and draw the Line A M ſo that it may onely touch the Arch L. Then ſetting one foot of the Compaſſes in I, with the other take the neareſt diſtance to the Line A M: this diſtance ſet from A to K, and draw the Line N K parallel to A B. So ſhall the diſtance B N, meaſured upon the Line of Chords, contain 18 degr. 47 min. the quantity of the enquired Side B E.

Case II.

The Side A B 30 *degr. the Angle at E* 38 *degr.* 15 *min. and the Side B E* 18 *degr.* 47 *m. being given, to finde the Angle at A.*

The Analogie or Proportion is,

As the Sine of the Baſe A B 30 degr. is to the Sine of the Angle at E 38 degr. 15 min.

So is the Sine of the Side B E 18 degr. 47 min. to the Sine of the Angle at A.

YOur Quadrant being drawn, from your Line of Chords take 30 degr. the quantity of the Side A B, and ſet them from B to E. Alſo take 38 degr. 15 min. the quantity of the Angle at E, and ſet them from B to F. Likewiſe take 18 degr. 47 min. the quantity of the given Side B E, from your Line of Chords, and ſet them from B to N; and draw the Lines E H, and F G, and N K, all parallel to A B.

This done, take in your Compaſſes the diſtance A H, and ſetting one foot in G, with the other deſcribe the Arch L, drawing the Line A M onely to touch the Arch. Then take the diſtance A K in your Compaſſes, and ſetting one foot of them upon the Line A C, move it along that Line gently, till the other Point, being turned about, will onely touch the Line A M; and where the Compaſs-point reſteth upon the Line A C, which it will doe at the Point I, through I therefore draw

the Line I D parallel to B A. So ſhall B D, being meaſured on your Line of Chords, contain 23 degr. 30 min. the quantity of the enquired Angle at A.

Case III.

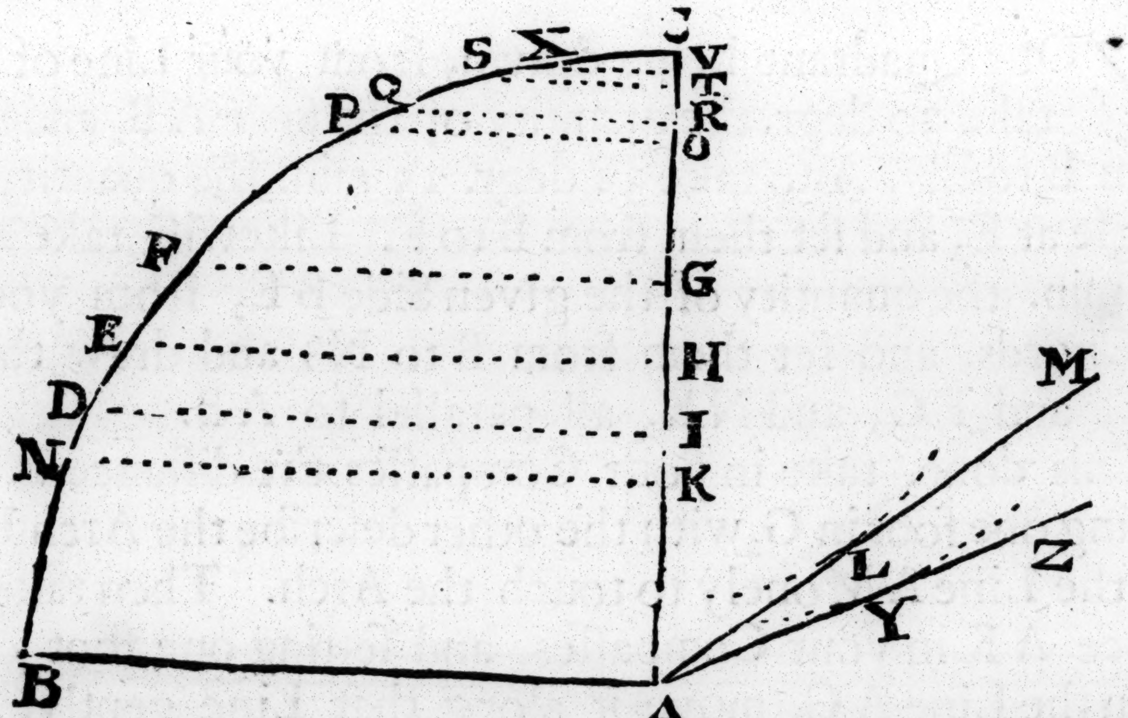
The Side A B 30 degr. and the Side B E 18 degr. 57 min. and the Angle at A 23 degr. 30 min. being given, to finde the Baſe A E.

The Analogie or Proportion is,

As the Co-ſine of A B 60 degr. is to the Co-ſine of B E 71 d. 13 min.

So is the Co-ſine of A C 62 degr. 6 min. the place where a Perpendicular would fall from B, to the Co-ſine of C E 75 degr. 3 min. Which C E, being added to A C, will give the quantity of the Side A E.

HAving drawn your Quadrant, take 60 degr. the Complement of A B, out of your Line of Chords, and ſet



them from B to P. Alſo take 71 degr. 13 m. the Complement of

of B E, and ſet them from B to S. In like manner take 62 d. 6 m. and ſet them from B to Q; and draw the Lines P O, S T, and Q R, all of them parallel to B A.

This done, take in your Compaſſes the diſtance A O, and ſetting one foot in T, with the other deſcribe the Arch Y; and draw the Line A Z onely to touch the Arch Y. Then take the diſtance A R, and ſetting one foot upon the Line A C, move it gently along the ſame, till the other foot, being turned about, do onely touch the Line A Z; and where the Compaſs-point reſteth, which it will doe at V, through that Point draw a Line V X parallel to B A. So ſhall the diſtance C X, meaſured upon your Line of Chords, contain 14 degr. 57 min. which added to the former Segment of the Side A C 27 degr. 54 min. the Sum will be 42 degr. 51 min. the quantity of the whole Side A E, which was required.

CASE IV.

The Angle at A 23 degr. 30 min. the Angle at B 122 degr. 36 m. and the Side A B 30 degr. given, to finde the Angle at E.

By the foregoing Caſes in Rectangled Sphericall Triangles (if you let fall a Perpendicular from the Angle B upon the Side A E, as B C,) you may finde the Angle A B C 69 degr. 22 min. and the Angle C B E 53 degr. 14 min. which being obtained,

The Analogie or Proportion is,

As the Sine of the Angle A B C 69 degr. 22 min. is to the Sine of the Angle C B E 53 degr. 14 min.

So is the Co-ſine of A 66 degr. 30 min. to the Co-ſine of the Angle at E.

THE Angles being found and the Quadrant drawn, take 53 degr. 14 min. out of your Chord, and ſet them from B to E. Likewiſe take 69 degr. 22 min. and ſet them from B

to G. Alſo take 66 degr. 30 min. and ſet them from B to F; and draw the Lines G H, F I, and E K, all parallel to B A.

This done, take in your Compaſſes the diſtance between A and K, and ſetting one foot in H, with the other deſcribe the Arch M, and draw the Line A N onely to touch the Arch. Again, ſetting one foot of the Compaſſes in I, with the other take the leaſt diſtance to the Line A N, which diſtance ſet from A to L. Laſtly, draw the Line L D parallel to B A, cutting the Quadrant in D. So ſhall the Arch B D, meaſured upon your Chords, contain 51 degr. 45 min. the Complement of the enquired Angle at E; or, D C ſhall give you 38 degr. 15 min. the Angle it ſelf.

Case V.

The Angle at A 23 degr. 30 min. the Angle at E 38 degr. 15 m. and the Side A B 30 degr. to finde the Angle A B E.

By the preceding Caſes of Right-angled Sphericall Triangles, the Perpendicular B C being let fall, finde the Angle A B C, which will be found to be 69 degr. 22 min. This Angle being obtained,

The Analogie or Proportion is,

As the Co-ſine of the Angle at A 66 degr. 30 min. is to the Co-ſine of the Angle at E 51 degr. 45 min.

So is the Sine of A B C 69 degr. 22 min. to the Sine of the Angle C B E.

THE Angle A B C being found, and the Quadrant drawn, take 66 degr. 30 min. out of your Line of Chords, and ſet them from B to F. Alſo take 51 degr. 45 min. from the Chord, and ſet them from B to D. Likewiſe take 69 degr. 22 min. (the quantity of the Angle found A B C) from your Chord,

Chord, and ſet them from B to G, and draw the Lines F I, D L, and G H, all parallel to A B.

This done, take in your Compaſſes the diſtance A L, and ſetting one foot of that extent in I, with the other deſcribe the Arch M, and draw the Line A N onely to touch the Arch. Again, ſet one foot of the Compaſſes in H, and with the other take the leaſt diſtance to the Line A N, which had, it will reach from A to K; through which Point K draw the Line K E parallel to B A, cutting the Quadrant in E. So ſhall the diſtance B E, being meaſured on your Line of Chords, contain 53 degr. 14 min. the quantity of the remaining part of the Angle A B E; which being added to the Angle A B C (before found) 69 degr. 22 min. the Sum of them will be 122 degr. 36 min. the quantity of the Angle A B E, which was required.

These Five laſt Caſes are all that in Oblique-angled Sphericall Triangles are reſolvable by Sines onely, except the Three laſt, which we reſolve by another kind of Artifice. The 6. 7. 8. *and* 9. *following have Tangents ingredient in the Proportion, and ſo do require a different way of reſolving, which is as followeth.*

Case VI.

The Side A B 30 *degr. the Baſe A E* 42 *degr.* 51 *min. and the Angle at A* 23 *degr.* 30 *min. being given, to find the Angle at E.*

The Perpendicular B C being let fall from the Angle at B, you muſt find the Segment of the Baſe C E 14 degr. 57 min. by the preceding Caſes of Right-angled Sphericall Triangles: Which had,

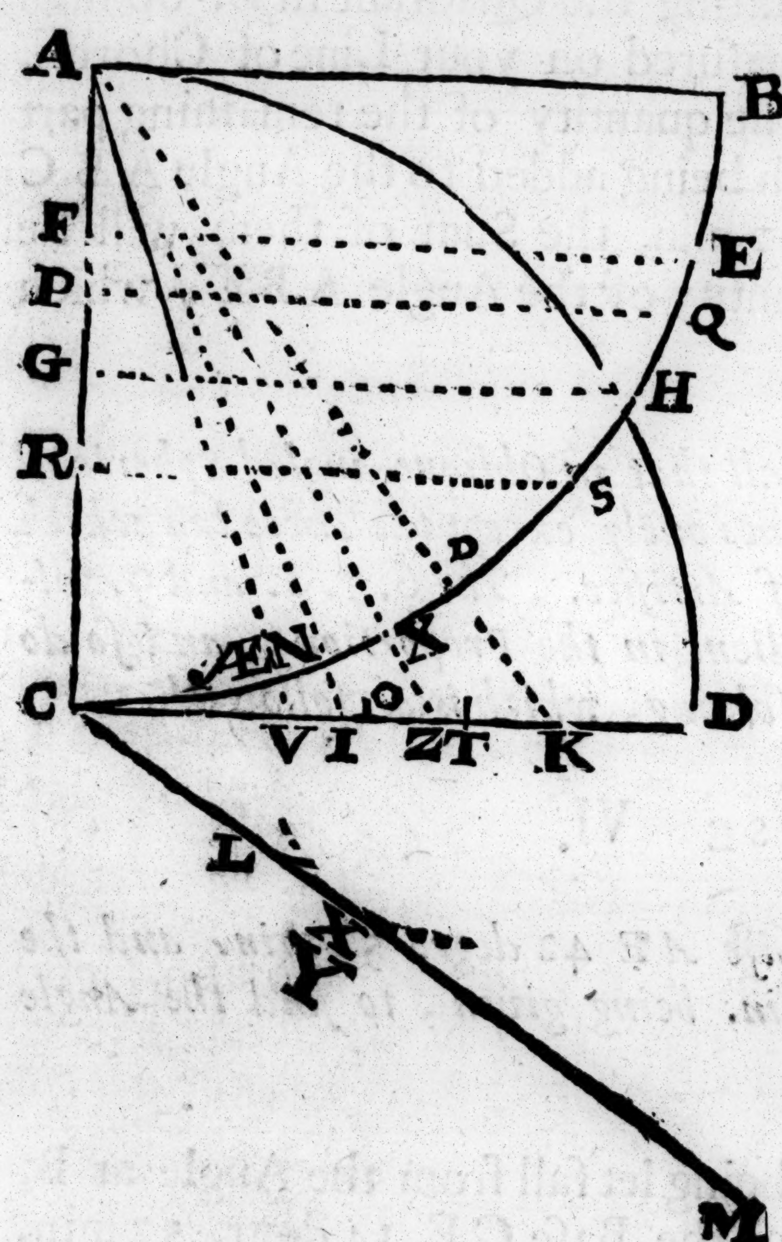
The Analogie or Proportion is,

As the Sine of the Side C E found 14 degr. 57 min. is to the Sine of A C 27 degr. 54 min.

So

So is the Tangent of the Angle at A 23 degr. 30 min. to the Tangent of the Angle at E.

YOur Quadrants A B C and C A D being drawn, take 14 degr. 57 min. the quantity of C E found, out of your Line of Chords, and ſet them from B to E. Alſo take 27 d. 54 min. the Segment of the Baſe A E, and ſet them from B to H; and draw the Lines E F and G H both parallel to A B. Alſo, take out of your Line of Chords 23 d. 30 min. the quantity of the Angle at A, and ſet them from C to N; and draw the Line A N, continuing it to I.



This done, take the diſtance A G, and ſet it from C to O. Alſo take the diſtance A F, and ſetting one foot in O, with the other deſcribe the Arch L; and draw the Line C M only that it may touch the Arch L. Then take in your Compaſſes the diſtance C I, ſet one foot upon the Line C D, moving it along the ſame till the other, being turned about, do onely touch the Line C M; and where the other Point of the Compaſſes reſteth upon the Line C D, which you will find it to doe at K, draw the Line A K, cutting the Quadrant in P. So ſhall C P, being

being meaſured upon your Line of Chords, contain 38 degr. 15 min. the quantity of the Angle E, which was required to be found.

Case VII.

The Angle at A 23 degr. 30 min. the Angle at B 122 degr. 36 m. and the Side A B 30 degr. being given, to find the Side B E.

Having let fall the Perpendicular B C, and found the Angle B C E by the former Caſes, then

The Analogie or Proportion is,

As the Co-ſine of C B E 38 degr. 46 min. is to the Co-ſine of A B C 20 degr. 38 min.
So is the Tangent of A B 30 degr. to the Tangent of B E.

THE Quadrants drawn, out of your Line of Chords take 38 degr. 46 min. the Complement of the Angle C B E, and ſet them from B to S. Alſo take from your Chords 20 degr. 38 min. the Complement of the Angle A B C, and ſet them from B to Q, and draw the Lines P Q and R S both parallel to C D. Again, take in your Compaſſes the diſtance A R, and ſet it from C to T. Likewiſe take the diſtance from A to P, and ſetting one foot of that diſtance in T, with the other deſcribe the Arch Y, and by it draw the Line C M onely to touch the Arch Y. Then take 30 degr. from your Chord, and ſet them from C to X, drawing the Line A X, and prolonging it to Z. Laſtly, from the Point T take the neareſt diſtance to the Line C M, which diſtance ſet from C to V, and draw the Line A V, cutting the Quadrant in Æ. So ſhall C Æ contain 18 degr. 47 min. of your Line of Chords, which is the quantity of the enquired Side B E.

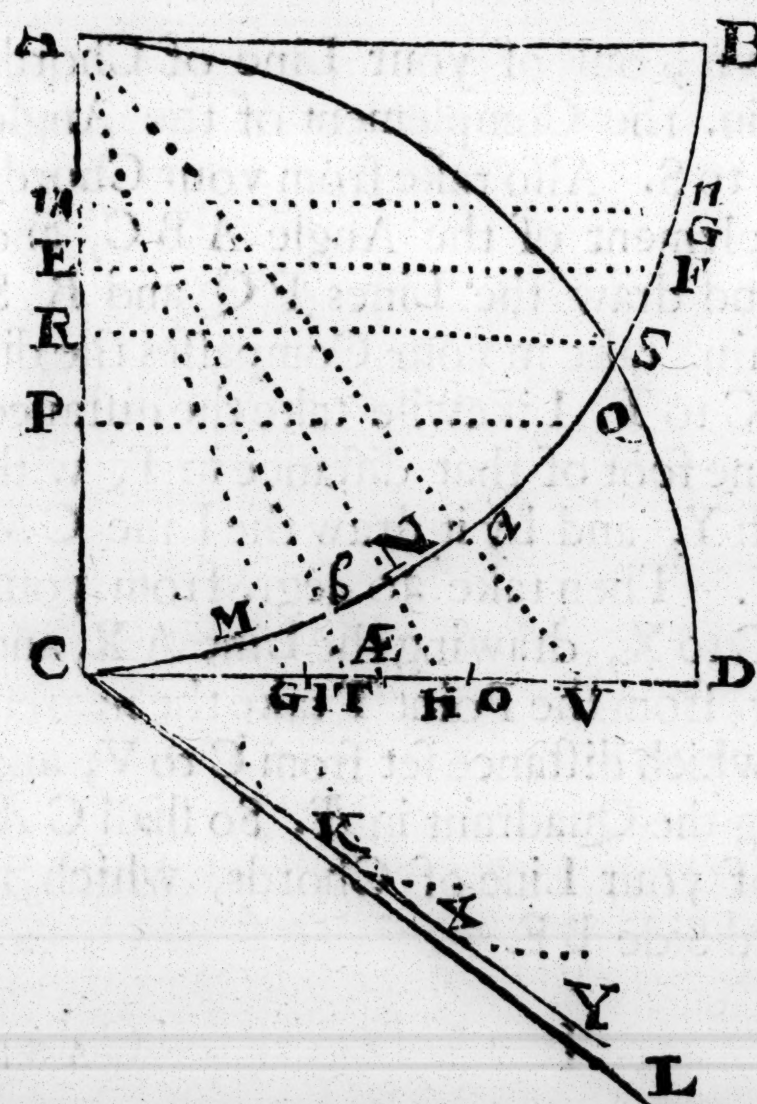
Case VIII.

The Side A B 30 *degr. the Side* B E 18 *degr.* 47 *min. and the Angle at A* 23 *degr.* 30 *min. being given, to finde the Angle at B.*

Having let fall the Perpendicular B C, and found the Angle A B C 69 degr. 22 min. by the foregoing Cases of Right-angled Sphericall Triangles,

The Analogie or Proportion is,

As the Tangent of B E 18 degr. 47 min. is to the Tangent of A B 30 degr.
So is the Co-sine of A B C 20 degr. 38 min. to the Co-sine of C B E.



HAVING drawn your Quadrants, out of your Line of Chords take 18 degr. 47 min. the Side B E, and set them from C to M. Also take 30 d. the quantity of the Side A B, from your Chord, and set them from C to N; and draw the Lines A M and A N, continuing them to G and H. Likewise take 20 degr. 38 min. out of your Line of Chords, and set them from B to F, and draw F E parallel to A B.

This

This done, take in your Compaſſes the diſtance C G, and ſetting one foot in the Point H, with the other deſcribe the Arch K, and draw the Line C L onely to touch it. Then take in your Compaſſes the diſtance A E, and ſetting one foot of the Compaſſes upon the Line C D, move it along the ſame gently, till the other, being turned about, do onely touch the Line C L; and where the Compaſs-point reſteth, which you will finde it to doe at O, make a mark, and take the diſtance O C in your Compaſſes, and ſet it from A to P, and draw the Line P Q parallel to A B. So ſhall B Q, meaſured upon the Line of Chords, contain 36 degr. 46 min. the Complement of the Angle C B E; or, the diſtance C Q, meaſured upon the Chords, will give 53 degr. 14 min. the Angle C B E it ſelf; which being added to 69 degr. 22 min. the Angle A B C, the Sum will be 122 degr. 36 min. the quantity of the whole Angle A B E required.

CASE IX.

The Angle at A 23 degr. 30 min. the Angle at E 38 degr. 15 m. and the Side B A 30 degr. being given, to find the Side E A.

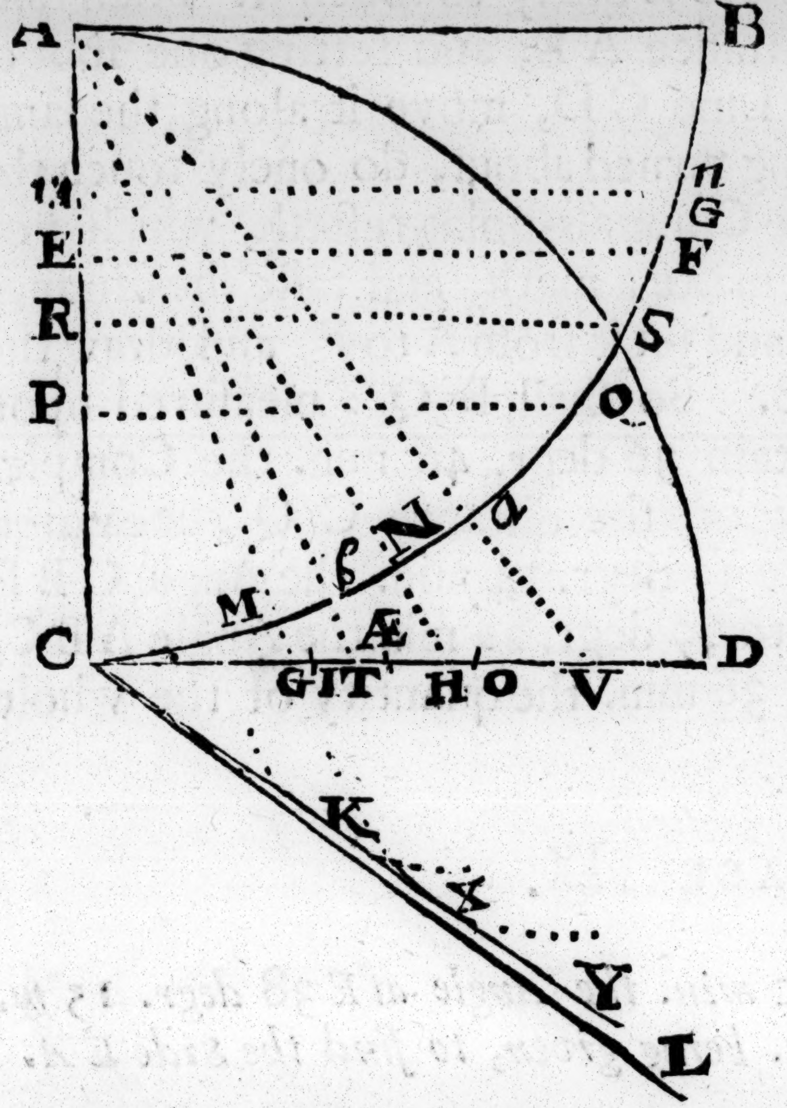
The Perpendicular being let fall, and A C 27 degr. 54 min. a part of the Side A E; then

The Analogie or Proportion is,

As the Tangent of the Angle at E 38 degr. 15 min. is to the Tangent of the Angle at A 23 degr. 30 min.
So is the Sine of A C 27 degr. 54 min. to the Sine of E C.

HAving drawn your Quadrants, take out of your Line of Chords 38 degr. 15 min. the quantity of the Angle at E, and ſet them from C to *a*. Alſo take 23 degr. 30 min. the quantity of the Angle at A, and ſet them from C to *b*, drawing the Lines A *b* and A *a*, and continuing of them to T and V.

This done, take in your Compasses the distance C T, and setting one foot of them in V, with the other describe the Arch X, and draw the Line C Y onely to touch the Arch X. Then take in your Compasses the distance A R, and set it from C to Æ, and from the Point Æ take the least distance to the Line C Y, which distance set from A to *m*, and draw the Line *m n* parallel to A B. So B *n*, being measured upon your Line of Chords, shall contain 14 degr. 57 min. a portion of the Side A E; which being added to the other portion A C 27 degr. 54 min. the Sum of them is 42 degr. 51 min. the whole Side A E, which was required.



Case X.

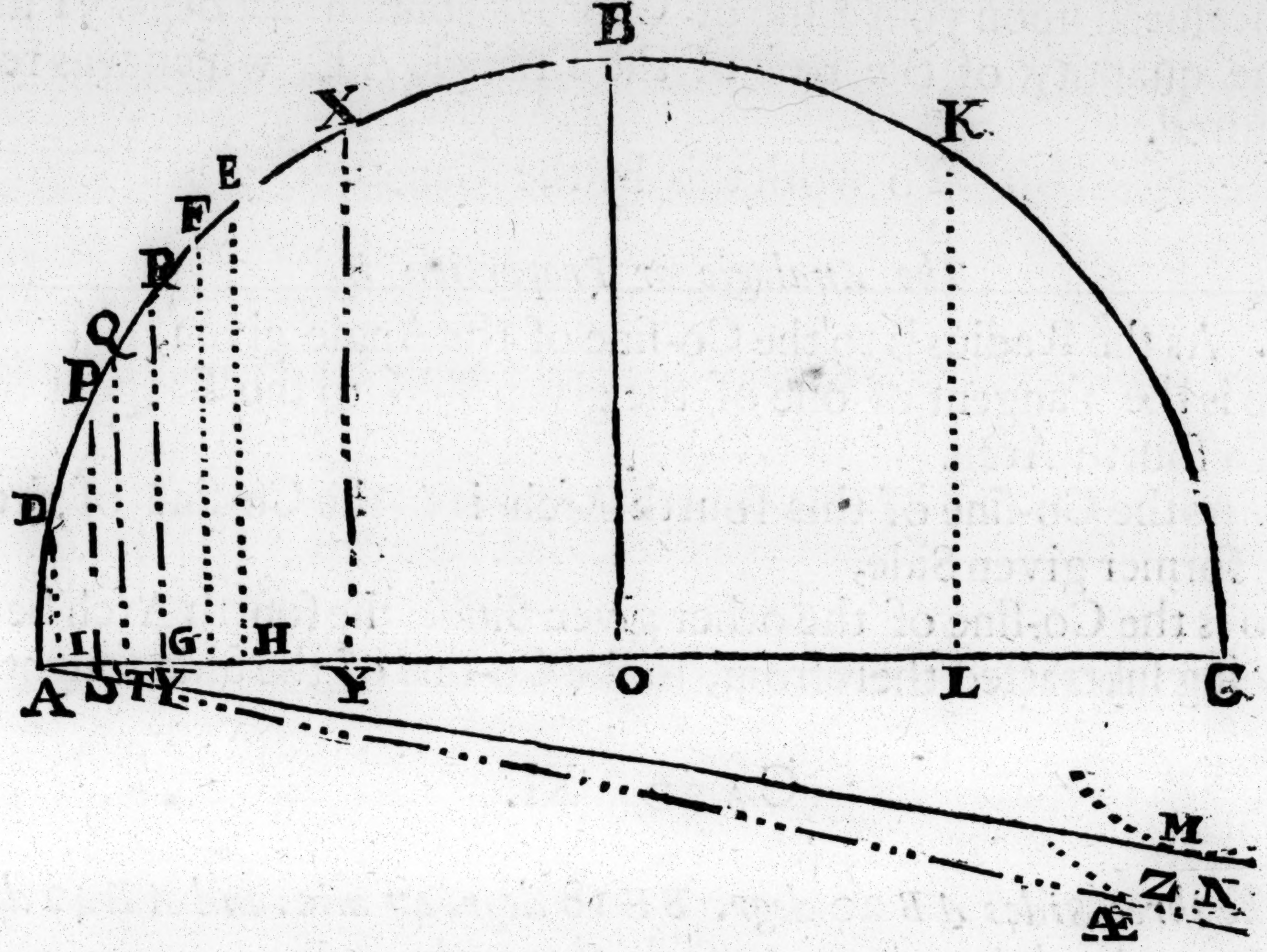
The Side A B 30 degr. the Side B E 18 d. 47 m. and the Angle at B 122 d. 36 m. contained by them, being given, to find the Base.

Take the Sum and the Difference of the two Sides A B 30 d. and B E 18 degr. 47 min.

	degr.	m.
Their Sum is	48	47
Their Difference is	11	13

First,

FIrst, draw a right Line A O C, and upon the Centre O describe the Semicircle A B C, drawing the Semidiameter or Perpendicular B O.



Being thus prepared, take out of your Line of Chords 48 d. 47 min. the Sum of the two Sides given, and set them upon the Semicircle from A to E. Also take 11 degr. 30 min. the Difference of the two given Sides, and set them from A to D. Again, take 122 degr. 36 min. the quantity of the given Angle, and set them from A to K; [but A B being 90 degr. set the residue, namely, 32 degr. 26 min. from B to K,] and draw the Lines D I, E H, and K L, all of them perpendicular to the Line A C.

This done, take in your Compasses the distance between I and H, and setting one foot of that extent in C, with the other describe the Arch M, and draw the Line A N so that

t may onely touch the Arch M. Then ſetting one foot of the Compaſſes in L, with the other take the neareſt diſtance you can from L to the Line A N, and ſet that diſtance from I to G. Laſtly, upon the Point G erect the Perpendicular G F, cutting the Semicircle in F. So ſhall the diſtance A F, being meaſured upon your Line of Chords, contain 42 degr. 51 m. the quantity of the Baſe of the Triangle A E, which was required.

To work this by the Canon.

The Analogie or Proportion is,

1. As the Radius is to the Co-ſine of the Angle given,
So is the Tangent of one of the given Sides to the Tangent of a fourth Arch.
2. As the Co-ſine of this fourth Arch is to the Co-ſine of the former given Side,
So is the Co-ſine of the other given Side, the fourth Arch being ſubtracted therefrom, to the Co-ſine of the Side ſought.

Case XI.

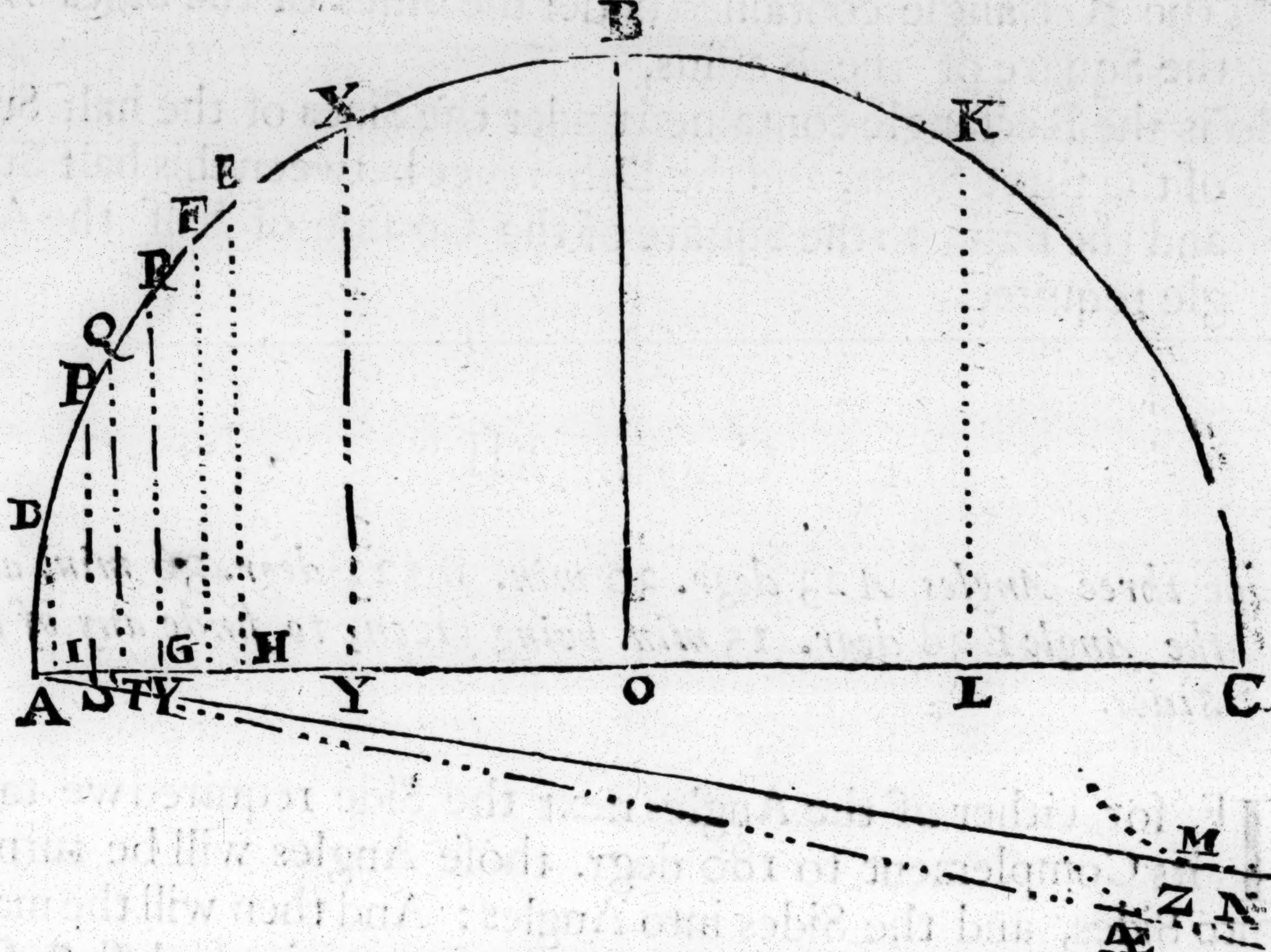
The three Sides AB 30 degr. BE 18 degr. 47 min. and AE 42 d. 51 min. being given, to finde the Angle at E.

Firſt, find the Sum and the Difference of the Sides B E and A E.

	degr.	m.
Their Sum is	61	38
Their Difference is	24	04

THE Sum and Difference of the two Sides being taken, out of your Line of Chords take 24 degr. 4 min. the Difference of them, and ſet them from A to P. Alſo from your Line of Chords take 61 degr. 38 min. the Sum of the two Sides, and ſet them from A to X. Again, take 30 degr. the

the quantity of the third Side A B, and set them from A to Q; and draw the Lines P S, Q T, and X Y, all three perpendicular to A C.



This done, take the distance between Y and S, and setting one foot of the Compasses in C, with the other describe the Arch Z, and draw the Line A Æ so that it may onely touch the Arch Z. Then take in your Compasses the distance between S and T, and setting one foot thereof upon the Line A C, move it gently along the same, till the other foot, being turned about, do onely touch the Line A Æ; and where the Compass-point resteth, which you will find it to doe at the Point Y, upon this Point Y erect the Perpendicular Y R. So shall the distance A R, being measured upon your Line of Chords, give 38 degr. 15 min. the quantity of the Angle at E, which was required to be found.

To work this by the Canon.

The Analogie or Proportion is,

As the Rectangle contained under the Sines of the Sides is to the Square of the Radius,

So is the Rectangle contained under the Sines of the half Sum of the three Sides, and the Difference between this half Sum and the Base, to the Square of the Co-sine of half the Angle required.

Case XII.

The three Angles A 23 *degr.* 30 *min. B* 122 *degr.* 36 *min. and the Angle* E 38 *degr.* 15 *min. being given, to finde any of the Sides.*

If for either of the Angles next the Side required we take its Complement to 180 degr. those Angles will be turned into Sides, and the Sides into Angles: And then will the manner of resolving the Triangle be the same as in the last *Case.*

Postscript.

Postscript.

BY *the* Precepts *before in this* Treatise *delivered you have the whole* Doctrine *both of* plain Right-lined Triangles, *and also of* Sphericall *both* Right *and* Oblique-angled, *wrought by a way not usually practised, which if it be carefully performed, you may by a* Line of Chords *of* 3 *or* 4 *inches* Radius *come within a few minutes of* Calculation; *which will be of sufficient use for any Practice either in* Astronomicall, Geographicall *or* Nauticall Questions, *wrought thereby. And if in the working of any of the* Cases *there be any difficulty, it is* in the moving of the Compasses along one Line, while the other foot, being turned about, doth only touch another Line. *But this is not a real, but a seeming, difficulty; for in* 3 *or* 4 *times trial you will finde it very expeditious and easie: and although it be not so Geometricall as the way by drawing of Parallels, yet it is altogether as exact; more ready and speedy in performance, and avoids the drawing of multiplicity of Lines, which would much cumber the Diagrams to no purpose. If the Schemes that are here drawn seem to be cumbred with multiplicity of Lines, the reason is, because many of them serve for* 3, 4, *or* 5 Cases; *but if a particular Diagram had been drawn for every* Case, *then none of them would have consisted of above* 5 *Lines besides the Quadrant, as you will see by trying of any* Case. *And here note, that every one of these* Cases, *both in* Right-lined *and* Sphericall Triangles, *is applicable to some one thing or other in the Practice of the* Mathematicks. *As, by* plain Right-lined Triangles *all Propositions are resolved that concern the taking of* Heights, Depths *and* Distances, Measuring *of* Land, Sailing *by the* plain Sea-Chart, *and also by* Mercator'*s*, *and divers other particulars. The Solution of* Sphericall Triangles *resolves Questions in* Astronomy, Geography, Dialling, Sailing by the Arch of a great Circle, *with many other particulars too tedious here*

to be recited; of some of which (especially those that concern the taking of Distances *in* Astronomy, Geography, Navigation, *&c.) I shall in the following Treatises exemplifie their Use.*

And because the Doctrine *of* Sphericall Triangles *is more difficult then that of* Plain Triangles, *I will in this next Treatise following shew you how (by your Line of Chords onely) you may* draw *or* project *the* Globe *or* Sphere *upon a* Plain; *by which means you may see before you how the* Sphericall Triangles *lie in the* Sphere *it self; which will be a great help to the understanding of the nature of a* Sphericall Triangle. *The* Sphere *being thus projected upon a Plain, I will shew how (upon that Plain) to measure both the* Sides *and* Angles *of a* Sphericall Triangle, *and, in so doing, resolve some Questions in* Astronomy; *by which you may discern, in that particular, how subservient* Trigonometry *is to* Astronomy, *and, as the Proverb is,* judge of *Hercules* by his Foot: *for to instance in the Variety of things that the Doctrine of* Triangles *is assistent to, were an endless Work.*

To *project* the SPHERE Upon the Plain of the *MERIDIAN* BY *A LINE OF CHORDS.*

Whereby the *Sides* and *Angles* of *Sphericall Triangles* are naturally laid down *in Plano*, as they are in the *Sphere* it self; By which the *nature* of them is discovered, and their *Sides* and *Angles measured* with *speed* and *exactness.*

The Fourth Exercise.

B*Eing to treat of the* Projection *of the* Sphere in Plano, *I suppose the Reader to be acquainted with the* Doctrine of the Sphere *or* Globe, *and with the* Circles *thereof; the* nature *of them, how they are there* situate *one from another in respect of distance, and to what* use *each of them is appropriate. But if there be any that hath a desire to make use of this*

Treatiſe, *and is ignorant of the* Sphericks, *let them reade the Books of ſuch as have written of the* Sphere *or* Globe. *In* Latin *there are divers, as* Theodocius, Orontius, Clarius, *&c. In* English *there is* Record's *Caſtle of Knowledge*, Hill's *School of Skill*, Blundevil's *Exerciſes*, Hews *of the Globes*, Newton, Moxon, *&c. But that this* Treatiſe *may not be accounted deficient in that which is ſo abſolutely neceſſary for the underſtanding and practice of what is herein contained; I will in this place give you a* brief *and* plain *Deſcription of the* Names, Properties, Diſtances, *&c. of ſuch* Circles *of the* Sphere *as in this Book we ſhall have occaſion to* project *for the delineating of* Sphericall Triangles in Plano, *and that in a correſpondent Poſition to their ſituation on the* Globe *or* Sphere *it ſelf.*

The Circles *therefore chiefly made uſe of in this* Projection *are theſe:*

1. *The* Meridian
2. *The* Horizon
3. *The* Æquinoctial
4. *The* Ecliptick
5. *The* Prime Verticall, *or* Circle of Eaſt and Weſt
6. *The* Hour-Circles
7. Azimuths, *or* Verticall Circles
8. *The two* Tropicks
9. Parallels *or* Circles *of* Declination
10. Circles *or* Parallels *of* Altitude.

Of theſe Circles *all but the three laſt are* great Circles *of the* Sphere, *which divide it into two equal parts; and the two* Tropicks, *the* Circles of Altitude *and* Declination, *are* ſmaller Circles, *and divide the* Sphere *into two parts unequally.*

Beſides theſe great *and* ſmall Circles, *there are ſeverall* Points *of note upon the* Globe: *as* (1.) *The* Zenith, *which is the Point in the Heavens directly over our heads, in what part of the Earth ſoever we be;* (2.) *The* Nadir, *which is the Point directly under our feet;* (3.) *The* Poles of the World, *about which the Heavens are moved;* (4.) *The Poles of the Ecliptick;* (5.) *The Poles of all other Circles.*

I. *Of the* MERIDIAN.

THE *Meridian* is a great Circle of the *Sphere*, which passeth through both the *Poles* of the *World*, and also through the *Zenith* and *Nadir Points*, and crosseth the *Horizon* in the *North* and *South* Points thereof. Unto this Circle (any Day in the year) when the Sun cometh it is Noon or Mid-day; and when the Moon, Stars or Planets, in the Night come to touch this Circle, they are then said to be upon the *Meridian*, or at the highest they will be that Night. This Circle in the Scheme of this Projection is noted by the Letters Z H N O.

II. *Of the* HORIZON.

THE *Horizon* also is a great Circle of the *Sphere*, and it is that Circle which divideth the visible part of the Heavens which we see from the not visible, that is, it divideth the *Sphere* into two *Hemispheres*, the lower and the higher. To this Circle when either the Sun, Moon, Stars or Planets, come on the *East* part, they are then said to *rise*; and when they have passed from the *Easterly* Point, by the *Meridian*, and descended to the *Western* part of this Circle, they are then said to *set*. This Circle is represented in the Projection by the right Line H A O.

III. *Of the* ÆQUINOCTIAL.

THE *Æquinoctial* is a great Circle, and in the *Sphere* it is elevated above the *Horizon* (upon the *Meridian* Circle) so much as is the Complement of the Latitude of the Place. As at *London*, where the Latitude is 51 degr. 30 min.

there the *Æquinoctial* is elevated 38 degr. 30 min. (which is ſo much as 51 degr. 30 min. wants of 90 degr.) and it cutteth the *Horizon* in the Points of *Eaſt* and *Weſt*. Unto this Circle when the Sun cometh (which is twice every year, namely, about the 10. of *March* and the 12. of *September*) it cauſeth the Daies and Nights to be of equal length all the World over. This Circle is noted in the Scheme with Æ A æ, and cuts the *Horizon* in the Point A, which repreſents both the *Eaſt* and *Weſt* Points thereof.

IV. *Of the* ECLIPTICK.

THIS alſo is a great Circle of the *Sphere*, and (in the *Northern Hemiſphere*, where the *North Pole* is viſible above the *Horizon*, and the *South Pole* not viſible) is elevated above the *Æquinoctial* Circle ſo much as is the Sun's greateſt Declination, which is 23 degr. and about 30 min. and is as much depreſſed below the *Æquinoctial* in the *Southern Hemiſphere*. This Circle is called by ſome *The Way of the Sun*, for that the Sun in his motion never ſwerveth or goeth out thereof, and ſo his *Longitude* or *Place* is counted in this Line. It cutteth the *Horizon* in the *Eaſt* and *Weſt* Point A, as the *Æquinoctial* did. It is repreſented in the *Scheme* by the Line ♋ A ♑, and hath charactered upon it the 12 Signs of the Zodiack; the *ſix Northern Signs*, ♈ ♉ ♊ ♋ ♌ and ♍ being on that half which is above the *Horizon*, and the *ſix Southern Signs* ♎ ♏ ♐ ♑ ♒ and ♓, on the other half, which is below the *Horizon*.

V. *Of the* PRIME VERTICALL.

THE *Prime Verticall*, or Circle of *Eaſt* and *Weſt*, (generally called the *Æquinoctial Colure*, and then (as the *Sphere* is here projected) the *Meridian* repreſenteth the *Solſtitial*

Colure)

Colure) is a great Circle passing through the *Zenith* and *Nadir* Points, and also through the *East* and *West* Points of the *Horizon*. Unto this Circle when the Sun, Moon, Stars or Planets, do (in their Motions) arrive, they are then due *East* or *West*. It is in the Projection signified by the right Line Z A N, passing through Z, the *Zenith*, N, the *Nadir*; and A, the *East* and *West* Point of the *Horizon*: and also cutteth the *Æquinoctial* in the Points ♈ and ♎.

VI. *Of the HOUR-CIRCLES.*

THE *Hour-Circles* are great Circles of the *Sphere*, meeting together in the *Poles* of the World, and crossing the *Æquinoctial* at right Angles, dividing it at every 15 degrees; and then every of those Divisions is one Hour of time: but if they pass through other parts of the *Æquinoctial*, dividing it unequally, then do those *Hour-Circles* represent unequal Spaces of time, according to the distance they are from the *Meridian*, or one from another. Of these Circles in the Scheme of the Projection there are four, thus noted P B S, P A S, P C S, and P D S.

VII. *Of the AZIMUTH CIRCLES.*

THE *Azimuth* or *Verticall Circles* are great Circles of the *Sphere*, meeting together in the *Zenith* and *Nadir* Points, as the *Hour-Circles* do in the *Poles* of the World, and divide the *Horizon* at right Angles, either equally, or unequally, as the *Hour-Circles* did the *Æquinoctial*. In the Scheme of the Projection there are four of these *Verticall Circles*, thus noted, Z ☉ N, Z F N, Z A N, and Z G N.

VIII. *Of*

VIII. *Of the* TROPICKS.

THE *Tropicks* are lesser Circles of the *Sphere*, dividing it unequally, and are drawn parallel to the *Æquinoctial*, at 23 degr. 30 min. distance therefrom, equal to the Sun's greatest Declination on either side. That *Tropick* which is on the *North-side* is called *The Tropick of Cancer*, to which when the Sun cometh (which is but once in a year, about the 10. of *June*) it maketh the *longest Daies* to all the *Northern* Inhabitants of the World, and the *shortest Nights*. The other *Tropick*, which is on the *South-side* of the *Æquinoctial*, is called *The Tropick of Capricorn*, to which when the Sun cometh, which is about the 11. of *December*, it maketh the *shortest Daies* and the *longest Nights* to all *Northern* Inhabitants, and the contrary to all the *Southern* Inhabitants of the World. In the Projection the *Tropick of Cancer* is signfied by ♋ I ♋, and the *Tropick of Capricorn* by ♑ K ♑.

IX. *Of the* CIRCLES *or* PARALLELS *of* DECLINATION.

THESE also are smaller Circles of the *Sphere*, and are drawn parallel to the *Æquinoctial*, towards both the *Tropicks*, and up to them. Those that are on the *North-side* of the *Æquinoctial* are called *Parallels of North Declination*, and those that are on the *South-side* of the *Æquinoctial* are called *Parallels of South Declination*. Of these *Parallels* there are in the Scheme of the Projection two, one towards the *Tropick of Cancer*, the other towards the *Tropick of Capricorn*, and either of them 20 degrees distant from the *Æquinoctial*. The *Northern Parallel of Declination* is noted with ♊ ☉ ♌, and the *Southern* with ♒ E ♐.

X. Of

X. *Of the* CIRCLES *or* PARALLELS *of* ALTITUDE.

THE Circles of *Altitude* are likewiſe ſmall Circles of the *Sphere*, and are drawn parallel to the *Horizon*, as the Circles of *Declination* were to the *Æquinoctial*. Theſe *Parallels* are drawn from the *Horizon* towards the *Zenith* Point, and upon occaſion, in many Caſes, quite up unto it. By theſe *Parallels* are meaſured the *Altitude* or *Height* of the Sun, Moon and Stars. In the Scheme there is onely one of them, and that is expreſſed by the Letters M E L.

Thus have I given you a brief and plain Deſcription *of the* Circles, *both* great *and* ſmall, *which we ſhall have occaſion to uſe in this following* Treatiſe. *And here note, that every* Circle *of the* Sphere (*both great and ſmall*) *hath his proper* Poles, *which* Poles (*of all the* great Circles) *are* 90 *Degrees, or a* Quadrant *of a* Circle, *diſtant from the* Circle *it ſelf. The* Poles *of the* Circles *in this* Projection *are as followeth.*

Z and N	Are the Poles of	H A O, the Horizon.
P and S		Æ A æ, the Æquinoctial.
O and H		Z A N, the Prime Verticall.
Q and R		the Ecliptick.
Æ and æ		P A S, the Axis of the World.

The Poles *of theſe five* Circles *are all in the* Meridian, *and ſo there needeth no farther Precept for the finding of them*; *and the* Pole *of the* Meridian *is the Centre thereof.*

But for the three Azimuth Circles, *they fall in ſeveral Points of the* Horizon; *and the three* Hour-Circles *in certain Points in the* Æquinoctial. *How to finde which Points ſhall be ſhewed afterwards in due place.*

A	Is the Pole of	the Meridian,	Z H N O.
T		the Azimuth Circle,	Z F N.
G			Z ⊙ N.
⊙			Z G N.
X		the Hour-Circle	P B S.
Y			P D S.
V			P C S.

The Poles of the World *P and S are also the* Poles *of the* Tropicks *and of all the* Parallels of Declination. *And*

The Zenith *and* Nadir, *Z and N, are the* Poles *of all the* Parallels *of* Altitude.

Having sufficiently acquainted the Reader with the several Circles, Lines, Points *and* Poles, *belonging to every* Circle, *I will now proceed to my intended purpose; namely, to project (or lay down* in Plano*) all these* Circles, Lines, Points *and* Poles, *in their true Positions.*

How to project the Sphere *upon the Plain of the* Meridian.

FIrst, take 60 degr. of your Line of Chords, and with that distance upon the Point A (as a Centre) describe the Circle Z H N O, representing the *Meridian*, (within which Circle all the rest are to be projected) and cross it with the two Diameters H A O the *Horizon*, and Z A N the *Prime Verticall.*

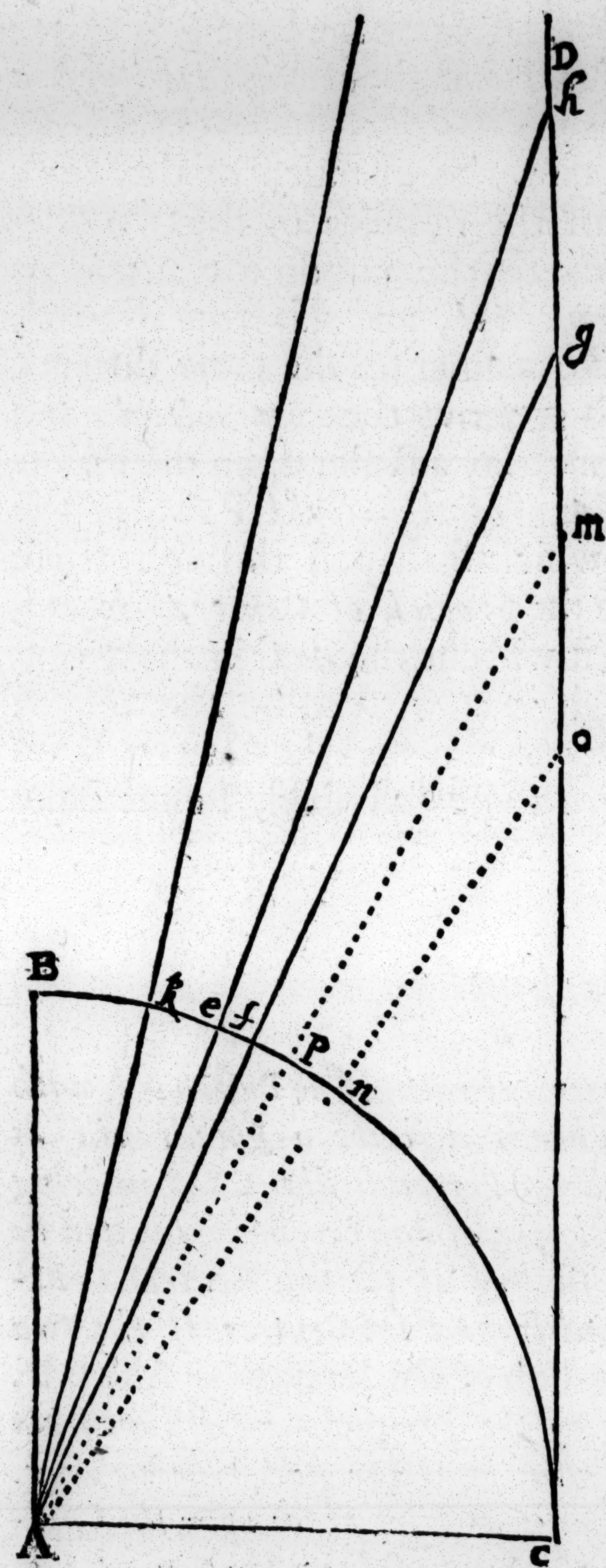
Secondly, (because the *Latitude* of the place for which you draw your *Projection*, viz. *London*, is 51 degr. 30 min.) take 51 degr. 30 min. from your Line of Chords, and set them upon the *Meridian* from Z to Æ, and from N to *æ*, and draw the

the Line Æ A *æ* for the *Æquinoctial.* Also set 51 degr. 30 m. from O to P, and from H to S, and draw the Line P A S, representing the *Axis of the World* and the *Hour-Circle of* 6 *a clock.*

Thirdly, take 23 degr. 30 min. the quantity of the *Sun's greatest Declination*, and also of the distance of the two *Tropicks* from the *Æquinoctial*, and set them upon the *Meridian* from Æ to ♋, above the *Æquinoctial*, and also from Æ to ♑, below the *Æquinoctial.* In like manner set the same distance of 23 degr. 30 min. from *æ* to ♋ above the *Æquinoctial*, and from *æ* to ♑ below it. This done, lay a Ruler upon the Points Æ and ♋, and it will cut the *Axis of the World* P A S in the Point I. So a Circle drawn which shall pass through these three Points, ♋ I ♋, shall be the *Tropick of Cancer.* Again, lay a Ruler to Æ and ♑, and it will cut the *Axis* in the Point K. So a Circle drawn through ♑ K ♑ shall be the *Tropick of Capricorn.* But to shew how you may find the Centres upon which these Tropical Circles are to be described, I must make this

Diversion.

Upon a long piece of stiff Paper, or rather fine Pastboard, with 60 *Degrees of your Line of Chords describe a Quadrant, as A B C, and upon the Point C erect the Perpendicular C D; in doing whereof you must be very carefull, for a small errour committed in that will produce a great one in finding of the true Centres.—Being thus prepared, you may readily find the Centres of the two* Tropicks, *and of any other* Parallels *of* Declination *or* Altitude. *But* (1.) *for the* Tropicks: *Being the* Tropicks *are distant from the* Æquinoctial 23 *degr.* 30 *min. on either side, take* 23 *degr.* 30 *min. out of your Line of Chords, and set them upon the* Quadrant *from B to f, and through the Point f draw the Line A f g, cutting the Perpendicular C D in the Point g. This done, take*



in your Compasses the distance C g, *and setting one foot in the Point* I *in your Projection, where the other Point falleth upon the Line* I P, *it being sufficiently extended, will be the Centre of the* Tropick of Cancer; *and a Circle described with this distance of the Compasses,* C g, *will pass exactly through the Points* ♋ I ♋, *and so describe your* Tropick of Cancer. *The like you must doe for the* Tropick of Capricorn, *by taking the distance* C g *in your Compasses, and setting one foot in the Point* K *of your Projection, and where the other reacheth upon the Line* K S, *being extended, there is the Centre of the* Tropick of Capricorn. *But* (2.) *to finde the Centres of any of the other* Parallels of Declination, *as of the two in the Projection, namely,* ♊ a ♌ *and* ♒ c ♐, *either of which are* 20 *d. distant from the Æ*-quinoctial; *the Centres of them are also found in the same manner as the Centres of the* Tropicks *were. For take* 20 *degr. out of your*

Line

Line of Chords, and set them upon the Quadrant *from B to e, drawing the Line A e h, the distance C h shall be the* Semidiameter *of the Parallel of* 20 *degr. of* Declination; *which, being set upon your* Projection *from a, upon the Line a P, (being extended) will there give you the Centre of the* Parallel; *and the Compasses at the distance of C h, being there set, will describe the* Parallel *of* 20 *degr. of* Declination ♊ *a* ♌. *And in the same manner that the Centre of this* Parallel *was found, so may you find the Centre of that on the other side of the* Æquinoctial, ♒ *c* ♐. *But* (3.) *you have in your* Projection *a* Parallel *or* Circle *of* Altitude, *namely, M E L, which is* 12 *degr. from or above the* Horizon, *the Centre whereof is found in the same manner as were the* Parallels *of* Declination. *For if you set* 12 *degr. of your Chord upon the* Quadrant *A B C, they will reach to k: draw a Line therefore from A through k, extending it till it meet with the Line C D being also extended; so shall the distance between this Point, where the two Lines meet, and the Point C, be the* Semidiameter *of the* Parallel *of* Altitude *of* 12 *degr. Wherefore that distance set from the Point m of your* Projection, *upon the Line m Z, being extended, will be the Centre of the* Parallel, *and the Compasses, at that distance, will describe the* Parallel *of* Altitude *M m L.*

But for those Parallels *of* Altitude *which fall near the* Horizon, *those* Circles *or* Parallels *of* Declination *which fall near to the* Æquinoctial, *those* Hour-Circles *which fall near to the* Axis of the World *or* Hour *of* Six, *and those* Azimuth Circles *which are near to the* Prime Verticall *or* Azimuth of East *and* West; *those that make* Mathematicall Instruments *have an Instrument called a* Bow, *which, by the help of one or more Screws, (according to the length of the* Bow) *may be extended to touch any three Points which lie near in a straight Line; by the edge of which* Bow *you may draw your* Hour-Circles, Azimuths, Parallels of Declination *and* Altitude, *as easily as you may draw a right Line by the edge of a Ruler.* ——— *But to return again to our* Projection.

Fourthly, draw a right Line ♋ A ♑ between the two *Tropicks*, touching the *Tropick of Cancer* above the *Horizon* at ♋, and the *Tropick of Capricorn* below the *Horizon* at the Point ♑.—This Circle hath upon it the Characters of the 12 Signs of the *Zodiac*, which are to be put on in this manner.—Take 23 d. 30 min. out of your Line of Chords, and ſet them from P to Q, and from S to R: which Points, Q and R, are the two *Poles* of the *Ecliptick*. Then take 60 degr. from your Line of Chords, and ſet them from Q to 1, and from Q to 3. Alſo ſet the ſame diſtance from ♋ to 2, and from ♑ to 4.—This done, lay a Ruler to the Pole R and the figure 1, it will cut the *Ecliptick* in the Point ♊ and ♌: the Ruler laid to R and 2 will cut it in the Point ♉ and ♍; and laid to R and 4, in ♏ and ♓; and laid to R and 3, in ♐ and ♒. So have you the true Points for the Sun's entrance into every Sign. And if you would have every tenth degree of each Sign, divide every of the Spaces ♋ 1, 1 2, 2 Q, Q 4, 4 3, and 3 ♑, into three equal parts; ſo will each part contain 10 d. and a Ruler laid to each of them and the Point R ſhall give you the Points upon the *Ecliptick* anſwering to the 10. degr. of every Sign. And in the ſame manner may you (if your Projection be large) put on every Degree.

Fifthly, for the putting on of the *Hour-Circles*; conſider how far the Circle you are to put on is diſtant from the *Meridian*, and ſet ſo many degrees upon the *Meridian* from the *Æquinoctial*: a Ruler laid from Z to thoſe degrees will croſs the *Æquinoctial*, and through that Point in the *Æquinoctial* where the Ruler ſo croſſeth, the *Hour-Circle* will paſs.—Example: The *Hour-Circle* P B S, in this Projection, is diſtant from the *Meridian* 62 d. 46 m. wherefore take 62 d. 46 m. from your Chords, and ſet them from *a* to *b*; then laying a Ruler from Z to *b*, it will cut the *Æquinoctial* in B, through which Point the *Hour-Circle* of 62 d. 46 m. muſt paſs.—To find the Centre of this *Hour-Circle*, (and ſo of any other) repair to the former Scheme for finding of the Centres of the *Parallels* of *Altitude* and *Declination*;

clination; and (because this *Hour-Circle* is distant from the *Meridian* 62 degr. 46 min.) take 62 degr. 46 min. from your Line of Chords, and set them upon the Quadrant A B C, from C to *l*, and draw the Line A *l m*. So shall the Line A *l m* be the Semidiameter of the *Hour-Circle* P B S; which being taken in your Compasses, and set upon your Projection from B, upon the Line B Æ, (being extended,) shall there give you the Centre of that *Hour-Circle*. And in the same manner may the Centres of all the rest be found.

Sixthly, the *Azimuth Circles* are to be drawn upon the Projection, and the Centres of them found in all respects as the *Hour-Circles* were.——So the *Azimuth Circle* Z ⊙ N, being 56 degr. 40 min. from the *Meridian*, take 56 degr. 41 min. out of your Line of Chords, and set them upon the *Meridian* of your Projection from O to *d*; then laying a Ruler unto Z and *d*, it will cut the *Horizon* in the Point ⊙, through which the *Azimuth* of 56 degr. 41 min. Z ⊙ N, must pass.——Then, to find its Centre, repair to the former Scheme for finding of Centres, and upon the Quadrant A B C set 56 degr. 41 min. of your Chords, from C to *n*, and draw the Line A *n o*: so shall the Line A *n o* be the Semidiameter of the *Azimuth Circle* Z ⊙ N; which being taken in your Compasses, and set upon your Projection from ⊙, upon the Line ⊙ H, (being extended,) shall there give you the Centre of the *Azimuth Circle* Z ⊙ N. And in this manner may the Centre of any other *Azimuth Circle* be found.

And here note (I.) *That the Centres of all* Azimuth Circles *fall in the* Horizon *H A O*, *being extended where need is. The Centres of all the* Hour-Circles *fall in the* Æquinoctial *Line Æ A æ*, *being extended. The Centres of the* Tropicks *and* Parallels *of* Declination *fall in the* Axis of the World *P A S*, *extended. And the Centres of the Circles of* Altitude *fall in the* Prime Verticall *Circle Z A N*.

Note (II.) *That if the middle Point of any* Hour-Circle *do not fall just in the* Æquinoctial, *or any* Azimuth Circle *just in the*

the Horizon, *but on either side of them; then you may find the Centres by the* Geometricall Propositions *at the beginning of this Book; though there be other waies to find the Centres upon the* Projection *it self, which I omit, for that I would not cumber the Scheme with unnecessary Lines.*

Seventhly, Every Circle in the Projection hath its proper *Pole*, as was before intimated. Now for the finding of them, you are to note, that the *Pole* of every *great Circle* is 90 degr. or a *Quadrant* of a Circle, distant from the Circle it self, upon that Line which cutteth the Circle at right Angles. Thus the *Poles* of all the *Hour-Circles* are upon the *Æquinoctial*, and the *Poles* of all the *Azimuths* upon the *Horizon*.——Now if you would find the *Pole* of the *Hour-Circle* P D S, lay a Ruler upon P and D, and it will cut the *Meridian* Circle in *e*: then take 90 degr. of your Line of Chords, and set them from *e* to *f*, a Ruler laid from P to *f* will cut the *Æquinoctial* in Y: so is Y the *Pole* of the *Hour-Circle* P D S.

Lastly, The finding of the *Poles* of the *Azimuth Circles* is the same with the *Hour-Circles*. So if you would find the *Pole* of the *Azimuth Circle* Z G N, lay a Ruler upon Z and G, it will cut the *Meridian* Circle in *g*; then set 90 degr. of your Chord from *g* to *d*, so a Ruler laid from Z to *d* will cut the *Horizon* H A O in the Point ⊙, which Point ⊙ is the *Pole* of the *Azimuth Circle* Z G N. And thus have you found the *Poles* of one of the *Hour* and one of the *Azimuth* Circles. And by the same manner of Work you may find the *Poles* of all the rest. As

The Pole of the	Hour-Circle	P D S	will be found at	Y
		P C S		V
		P A S		Æ or æ
		P B S		X
	Azimuth Circle	Z G N	will be found at	⊙
		Z A N		H or O
		Z F N		T
		Z ⊙ N		G

The

The Poles of the	Horizon	H A O	are	Z and N, the Zenith and Nadir.
	Æquinoctial	Æ A æ		P and S, the Poles of the World.
	Ecliptick	♋ A ♑		Q and R.

Thus have I given you at large a plain and easie method how to project *the* Sphere *upon the Plain of the* Meridian Circle, *by help of the Line of Chords onely: Upon which* Projection, *by the intersection or crossing of the severall* Circles *thereof, are constituted divers* Sphericall Triangles; *some Right-angled, and others Oblique-angled. By the resolving of which* Triangles *variety of* Questions *appertaining to* Astronomie, Geographie *and* Navigation, *may (with speed and exactness) be resolved. But before I come to shew the manner of working* particular Questions *of any kind, it will be expedient that I shew you,* (1.) *how to measure or find the quantity of the* Sides *and* Angles *of a* Sphericall Triangle, *as they are here projected;* *and* (2.) *how to* project *or* lay down *an* Angle *or* Side *of any quantity that shall be required.*

I. *A* Sphericall Triangle *being projected, how to find the quantity of any* Angle *thereof.*

LAY a Ruler to the angular Point, and the extremity of the Sides containing the Angle, they being continued to Quadrants; and note where the Ruler cuts the *Meridian* or outward Circle; at both which places make marks upon the *Meridian*: the distance between those two marks, being measured upon your Line of Chords, shall give you the quantity of the Angle required.

Example I.

IN the Triangle P ⊙ O, in the Projection, let it be required to find the quantity of the Angle ⊙ P O.———First, lay a Ruler upon the angular Point P, and to the extreme ends of the Sides P ⊙ and P O, they being extended to Quadrants, which is, to that Circle which measures that Angle: (as the *Æquinoctial* measures all the Angles at P, the *Pole of the World*; the *Horizon* all the Angles at Z, the *Zenith*, &c.) So the Ruler laid from P to *æ*, will cut the *Meridian* in *æ*; and being laid from P to B, it will cut the *Meridian* in the Point *b*. The distance *b æ*, being taken in your Compasses and measured upon your Line of Chords, will be found to contain 62 degr. 46 min. which is the quantity of the Angle ⊙ P O.———But if upon the Point P you were to project an Angle to contain 62 degr. 46 min. then take 90 degr. of your Chords, and set them from P to *æ*, and through the Centre A draw the Line Æ A *æ*; then take 62 degr. 46 m. out of your Line of Chords, and set them from *æ* to *b*; and laying a Ruler from P to *b*, it will cut Æ A *æ* in the Point B: the Circle P B S being drawn, the Angle at P will contain 62 degr. 46 min.

Example II.

LET it be required to find the quantity of the Angle Z E P.———Lay a Ruler to ⊙, the *Pole* of the Circle Z E N, and the Point E, it will cut the *Meridian* Circle in M; from M set 90 degr. to *z*; a Ruler laid from ⊙ to *z* will cut the Circle Z E N (it being extended beyond the *Zenith* Z) at the Point ∫.

Again, Lay a Ruler upon Y, the Pole of the Circle P E S, and it will cut the *Meridian* Circle in *v*; set 90 degr. from *v* to

to *x* upon the *Meridian*; a Ruler laid from Y to *x* will cut the Circle PES in *y*.

This done, lay a Ruler from E to δ, and it will cut the *Meridian* in θ; alſo lay the Ruler from E to *y*, it will cut the *Meridian in* λ: the diſtance θλ, being taken in your Compaſſes and applied to your Line of Chords, will be found to contain 21 degr. 45 min. And ſuch is the quantity of the Angle Z E P.

Theſe two ſorts of Angles *are the moſt troubleſome to find their quantities, and therefore I have inſtanced in them. There are other* Angles *in the* Projection *which render their meaſures to the eye, without farther Inſtructions for finding their quantities.*

II. *A* Sphericall Triangle *being projected, to find the quantity of any* Side *thereof.*

A Ruler laid upon the Pole of the Circle which is to be meaſured, and to the extreme ends of the Side of the Triangle; note where the Ruler, ſo laid, cuts the *Meridian* at both ends of the Side: that diſtance, taken in your Compaſſes and meaſured upon the Line of Chords, will give you the quantity of the Side of the Triangle.

Example I.

LET it be required to find the Side E Z of the Triangle Z E P.———Lay a Ruler to ⊙ (the Pole of the Circle Z E N) and the angular Point E, it will cut the *Meridian* in M; and a Ruler laid to Z will cut the *Meridian* in Z. So the diſtance M Z, taken in the Compaſſes and meaſured upon the Line of Chords, will be found to contain 78 degr. And ſuch is the quantity of the Side Z E.

Example II.

LET it be required to find the Side ⊙ B of the Sphericall Triangle A ⊙ B.—Lay a Ruler upon X, the Pole of the Circle P B S, and the Point B, it will cut the Meridian Circle in *æ*.—Also lay a Ruler from X to ⊙, it will cut the Meridian in the Point ♌. The distance between *æ* and ♌, being taken and measured on the Line of Chords, will contain 20 d. And such is the quantity of the Side ⊙ B.

I could instance in divers other Examples *concerning the Measuring of the* Sides *and* Angles *of* Triangles *upon the* Projection; *but I here omit them, because in the resolving of the following* Propositions *they will come in practice, and the Manner of the performance is there plainly expressed: onely I deemed it convenient here to give some taste thereof, as a Preparative to that which followeth.——But before I come to shew the Manner of resolving of particular Questions in* Astronomie, Geography, *&c. I will declare the Variety of* Sphericall Problems *that will naturally arise out of every* Sphericall Triangle, *being projected.*

THE

The Variety Of *Sphericall* PROBLEMS

Naturally arising out of every *Sphericall Triangle*, both *Right* and *Oblique-angled*, and that are resolvable thereby, described as they are perspicuous to the Eye in the *Projection*.

The Fifth Exercise.

IN *the foregoing Part of this Book you have the* Doctrine *of* Plain *and* Sphericall Triangles Geometrically *performed. And in the Solution of* Right-angled Sphericall Triangles *there were* 16 Cases; *and in* Oblique-angled *there were* 12 Cases: *but the* 16 Cases *of* Right-angled Triangles *will by this* projective *way be reduced to* 5 Cases, *and the* 12 *of* Oblique-angled *will be reduced to* 6; *so that in both there will be but* 11 Cases, *whereas before there were* 28. *That this may appear plain to the Reader, I will make use of* two

Triangles *in the* Projection; *one whereof shall be* Right-angled, *as the* Triangle *P O* ☉, Right-angled *at O*; *and the other shall be the* Oblique-angled Triangle *Z E P*.——*The* Right-angled Triangle *is constituted by the Intersection of three* great Circles *of the* Sphere; *namely, of P O, an Arch of the* Meridian, ☉ *O, an Arch of the* Horizon, *and P* ☉, *the Arch of an* Hour-Circle. —*The* Oblique-angled Triangle, *Z E P*, *is constituted also of three Arches of* great Circles *of the* Sphere, (*as all* Sphericall Triangles *whatsoever are*;) *namely, of Z P, an Arch of the* Meridian, *P E, an Arch of an* Hour-Circle, *and Z E, an Arch of an* Azimuth Circle.

In the Right-angled Sphericall Triangle, *P* ☉ *O*,

The Side:
- *P O is the* Latitude of the Place.
- ☉ *O is the* Sun's Amplitude from the North.
- *P* ☉ *is the* Sun's distance from the Pole, *or the* Complement of his Declination.

The Angle:
- ☉ *P O is the* Hour from Midnight, *or from the* North part of the Meridian.
- *P* ☉ *O is the Angle of the* Sun's Position *at the time of the Question.*
- *P O* ☉ *is the* Right Angle.

The Parts of the Triangle *being declared, and of what* Circles *of the* Sphere *the* Sides *do consist*; *I will now come to the* Cases, *which* (*as I said before*) *are* 5 *in every* Right-angled Triangle. *So that any two parts of the* Triangle (*besides the* Right Angle) *being given, I will shew in every of the* 5 Cases *what parts may be found.*

I. *In*

I. *In a Right-angled Sphericall Triangle.*

Case I.

The Base and Perpendicular being given, to finde the other parts of the Triangle.

IN the Triangle P ⊙ O here is given P O, the Latitude, and ⊙ O, the Amplitude from the North part of the *Meridian*; by which you may find

3. { 1. P ⊙, *the Complement of the Sun's Declination.*
2. ⊙ P O, *the Hour from Midnight.*
3. P ⊙ O, *the Angle of the Sun's Position.* }

Case II.

The Hypotenuse and Perpendicular being given, to find the other parts.

IN the Triangle P ⊙ O here is given P O, the Latitude, and P ⊙, the Complement of the Sun's Declination; by which may be found

6. { 1. ⊙ O, *the Amplitude from the North.*
2. O P ⊙, *the Hour from Midnight.*
3. P ⊙ O, *the Angle of the Sun's Position.*
And if in stead of the Perpendicular there had been given the Base ⊙ O, you might then find
4. P O, *the Latitude.*
5. ⊙ P O, *the Hour from Midnight.*
6. P ⊙ O, *the Angle of the Sun's Position.* }

Case

Case III.

The Hypotenuse and an Angle being given, to find the other Parts.

HERE is given in the Triangle P ⊙ O, the Complement of the Sun's Declination, P ⊙ and ⊙ P O, the Hour from Midnight; by which may be found

6.
1. ⊙ O, *the Amplitude from the North.*
2. P O, *the Latitude.*
3. C ⊙ O, *the Angle of the Sun's Position.*

   And if in lieu of the Angle at P, the Angle at ⊙ P had been taken, then you might have found

4. ⊙ O, *the Amplitude from the North.*
5. P O, *the Latitude.*
6. ⊙ P A, *the Hour from Midnight.*

Case IV.

The Perpendicular, or Base, and either of the Angles given, to find the other Parts.

IN the Triangle P ⊙ O let there be given the Amplitude, ⊙ O, and the Angle of the Sun's Position, P ⊙ O; by which you may find

12.
1. P O, *the Latitude.*
2. ⊙ P O, *the Hour from Midnight.*
3. P ⊙, *the distance of the Sun from the Pole.*

   But if the Side given had been ⊙ O, and the Angle given ⊙ P O, then you might have found

4. P ⊙, *the Complement of the Sun's Declination.*
5. P O, *the Latitude.*
6. P ⊙ O, *the Angle of the Sun's Position.*

   But again, if ⊙ O and ⊙ P O had been given, then might be found

7. ⊙ P, *the Complement of the Sun's Declination.*
8. ⊙ O, *the Amplitude from the North.*

9. P ⊙ O,

9. P ⊙ O, *the Angle of the Sun's Position.*

And again, if P O and O ⊙ P had been given, we might then have also found

10. ⊙ P, *the distance of the Sun from the Pole.*
11. ⊙ O, *the Amplitude from the North.*
12. ⊙ P O, *the Hour from Midnight.*

Case V.

The Angles being given, to find the other Parts.

If the two Angles P ⊙ O and O P ⊙ be given, there may be found

3. { 1. P O, *the Latitude.*
2. ⊙ O, *the Amplitude from the North.*
3. P ⊙, *the Complement of the Sun's Declination.* }

Thus you see, that in this one Right-angled Sphericall Triangle, *by the several Parts given in these* five Cases, *there are* 30 Sphericall Problems *resolved; and so many are resolvable in every* Right-angled Triangle.

II. *In an Oblique Sphericall Triangle.*

In the Oblique-angled Sphericall Triangle, Z P E,

The Side {
Z P is the *Complement of the Latitude.*
P E is the *Complement of the Sun's Declination*, or his distance from the *North Pole.*
Z E is the *Complement of the Sun's Altitude.* }

The Angle {
E Z P is the *Sun's Azimuth from the North part of the Meridian.*
Z P E is the *Hour from Noon.*
Z E P is the *Angle of the Sun's Position.* }

N Case

Case I.

The three Sides being given, to find an Angle.

IN the Triangle Z E P, if there be given the Side E Z, the Complement of the Sun's Altitude, Z P, the Complement of the Latitude, and E P, the Sun's distance from the Pole, or the Complement of his Declination, you may find

3. {
1. E Z P, *the Sun's Azimuth from the North.*
2. Z E P, *the Angle of the Sun's Position.*
3. Z P E, *the Hour from Noon.*
}

Case II.

Two Sides and the Angle comprehended by them being given, to find the other Parts of the Triangle.

IF in the Triangle Z E P there be given the Side E Z and Z P, and the Angle between them E Z P, you may find

9. {
1. Z E P, *the Angle of the Sun's Position.*
2. Z P E, *the Hour from the South, or Noon.*
3. E P, *the Sun's distance from the Pole.*

But if the Sides Z P and P E, and the Angle Z P E between them, had been given, then might have been found

4. P E Z, *the Angle of the Sun's Position.*
5. E Z, *the Complement of the Sun's Altitude.*
6. E Z P, *the Azimuth of the Sun from the North.*

And if the Sides Z E and P E, with the Angle Z E P contained by them, had been given, there might be found

7. E Z P, *the Sun's Azimuth from the North.*
8. Z P, *the Complement of the Latitude.*
9. Z P E, *the Hour from Noon.*
}

Case III.

Two Angles, and a Side contained by them, being given, to find the other Parts.

If in the Triangle Z E P the Angle E Z P and the Angle Z P E, with the Side contained between them, Z P, be given, we may find

1. Z E, *the Complement of the Sun's Altitude.*
2. Z E P, *the Angle of the Sun's Position.*
3. E P, *the Complement of the Sun's Declination.*

   But if the Side E P, and the Angles ZEP & ZPO, had been given, then might be found

4. E Z, *the Complement of the Sun's Altitude.*
5. E Z P, *the Azimuth from the North.*
6. Z P, *the Complement of the Latitude.*

   And if the Side Z E, and the Angles P Z E and P E Z, had been given, then you might find

7. Z P, *the Complement of the Latitude.*
8. Z P E, *the Hour from Noon.*
9. P E, *the Sun's distance from the Pole.*

9.

Case IV.

Two Sides, with an Angle opposite to one of them, being given, to find the other Parts.

If there be given the Side Z P, the Side E P, and the Angle Z E P, there may be found

1. E Z, *the Complement of the Sun's Altitude.*
2. E Z P, *the Sun's Azimuth from the North.*
3. Z P E, *the Hour from the South.*

   But if the Side Z P, and the Side E P, with the Angle E Z P, had been given, then might be found

4. E Z, *the Complement of the Sun's Altitude.*

 5. Z E P,

5. Z E P, *the Angle of the Sun's Position.*
6. Z PE, *the Hour from Noon.*

And if there had been given E P, E Z, and E Z P, then might be found

7. Z P, *the Complement of the Latitude.*
8. Z P E, *the Hour from Noon.*
9. Z E P, *the Angle of the Sun's Position.*

18. In like manner, if the Sides P E and E Z, with the Angle Z P E, had been given, then might be found

10. Z P, *the Complement of the Latitude.*
11. E Z P, *the Azimuth from the North.*
12. Z E P, *the Angle of the Sun's Position.*

Again, if the Sides E Z and Z P, with the Angle Z P E, had been given, then would be found

13. P E, *the Sun's distance from the Pole.*
14. Z E P, *the Angle of the Sun's Position.*
15. E Z P, *the Sun's Azimuth from the North.*

Lastly, if the Sides E Z and Z P, with the Angle Z E P, had been given, then you might find

16. P E, *the Complement of the Sun's Declination.*
17. Z P E, *the Hour from Noon.*
18. E Z P, *the Azimuth from the North.*

Case V.

Two Angles, and a Side opposite to one of them, being given, to find the other Parts of the Triangle.

IN the Triangle Z E P, if there be given the Angles E Z P and Z P E, with the Side P E, there may be found

1. Z P, *the Complement of the Latitude.*
2. Z E, *the Complement of the Sun's Altitude.*
3. Z E P, *the Angle of the Sun's Position.*

But if there were given E Z P and Z P E, with the Side Z E, then might be found

4. Z P,

4. Z P, *the Complement of the Latitude.*
5. P E, *the Sun's distance from the Pole.*
6. Z E P, *the Angle of the Sun's Position.*

And if the Angles Z P E and Z E P, with the Side E P, were given, then might be found

7. Z P, *the Complement of the Latitude.*
8. P E, *the Hour from Noon.*
9. E Z P, *the Sun's Azimuth from the North.*

18. Again, if there were given the Angles Z P E and Z E P, with the Side Z P, you might then find

10. Z E, *the Complement of the Sun's Altitude.*
11. P E, *the Complement of the Sun's Declination.*
12. E Z P, *the Sun's Azimuth from the North.*

Also if there were given the Angles ZEP and EZP, with the Side Z P, you might find

13. Z E, *the Complement of the Sun's Altitude.*
14. P E, *the Complement of the Sun's Declination.*
15. Z P E, *the Hour from Noon.*

And lastly, if there were given the Angles Z E P and E Z P, with the Side P E, then might be found

16. Z E, *the Complement of the Sun's Altitude.*
17. Z P, *the Complement of the Latitude.*
18. Z P E, *the Hour from Noon.*

Case VI.

The three Angles being given, to find the other Parts.

IN the Triangle Z E P, if the three Angles E Z P, Z P E, and P E Z, be given, there may be found

3.
1. Z P, *the Complement of the Latitude.*
2. P E, *the Sun's distance from the Pole.*
3. Z E, *the Complement of the Sun's Altitude.*

Thus have you in these six Cases all the Varieties that will arise out of an Oblique-angled Sphericall Triangle, in the Conversion of

which Cases you may observe 60 Questions of the Sphere to be resolved; and so many are resolvable in every Oblique-angled Sphericall Triangle, and 30 in every Right-angled: So that in these two Triangles 90 Questions are resolved. For,

In a Right-angled Sphericall Triangle,

By the	First	Case are resolved	3	Sphericall Questions.
	Second		6	
	Third		6	
	Fourth		12	
	Fifth		3	
			In all 30.	

In an Oblique-angled Sphericall Triangle,

By the	First	Case are resolved	3	Sphericall Problems.
	Second		9	
	Third		9	
	Fourth		18	
	Fifth		18	
	Sixth		3	
			In all 60.	

PRO-

PROPOSITIONS ASTRONOMICALL, Usefull in the Practice of NAVIGATION:

Performed by the resolving of severall *Sphericall Triangles* upon the *Projection*.

The Sixth Exercise.

OF *the vast number of* Sphericall Questions *that are resolvable by the severall* Sphericall Triangles *that are (by the Intersections of the* great Circles *of* the Sphere) *constituted upon the* Projection, *seeing there may be* 30 *in every* Right-angled, *and* 60 *in every* Oblique-angled Sphericall Triangle *resolved; I have made choice onely of such as I conceived most necessary for* Sea-mens *use. Of which, some of them will be assistent to them in finding the* Latitude; *others for finding the* Sun's Altitude *upon some certain* Points

of the Compass, *and at some particular* Hours: *others will help them to find the* Variation of their Compass; *and some of them will help them to the* Hour of the Day and Night.

By the manner of Working of these Propositions *upon the* Projection, *the ingenious* Practitioner *may propose divers other* Questions *to himself, and* project *the* Sphere in Plano *sutable unto them*; *in the* Practice *and true* performance *whereof he will accumulate to himself both* Pleasure *and* Profit.

Besides the working of these Propositions *upon the* Projection, *I have to every of them added the* Analogie *or* Proportion, *by which they may be resolved by the* Canons *of Artificial* Sines *and* Tangents; *so that such as require more exactness then can be expected from* Instrumental performance, *may by those* Canons *work and resolve their* Questions *more accurately. Many more* Propositions *I could have added, but these (for the* Astronomicall *part) I deemed sufficient: such as concern* Geographie *and* Navigation *shall follow in convenient place.*

Prop. I.

The distance of the Sun *from the nearest* Æquinoctial Point (*either* Aries *or* Libra) *given, to find his* Declination.

IN the *Projection* this *Proposition* is to be resolved upon the *Right-angled Sphericall Triangle* A *k* ♒, right-angled at ♒: which Triangle is constituted by the Intersection of the Arches of three great Circles of the Sphere; namely, of A ♒, *an Arch of the Ecliptick*; A *k*, *an Arch of the Æquinoctial*; and of ♒ *k*, *an Arch of a great Circle passing through Q and R the Poles of the Ecliptick*, and cutting the *Ecliptick* at right Angles in ♒, which is the place of the *Sun* at the time of the *Question*.

In which Triangle you have given (besides the right Angle at

at ♒) (1.) the Side A ♒, 59 degr. which is the distance of the Sun from the nearest Æquinoctial Point *Libra*; (2.) the Angle ♒ A k, 23 degr. 30 min. which is the Angle that the *Ecliptick* makes with the *Æquinoctial*, and is alwaies equal to the greatest *Declination* of the Sun. Wherefore in this *Triangle* you having given the Base A ♒, and the Angle at the Base k A ♒, you may find the Perpendicular k ♒, the *Sun's Declination*, by the 11. Case of Right-angled Sphericall Triangles in the first Part of this Book. For which this is

The Analogie or Proportion.

As the Radius 90 degr. is to the Sine of the Sun's greatest Declination 23 degr. 30 min.
So is the Sine of the Sun's distance from the next Æquinoctial Point *Libra* 59 degr. to the Sine of the Sun's present Declination 20 degr.

To resolve the Triangle upon the Projection,

Lay a Ruler upon the Pole of the Circle R ♒ Q, (which is at ♉,) and the Point k, it will cut the Meridian in the Point l: Also lay a Ruler from ♉ to ♒, it will cut the Meridian in the Point ♑. So the distance l ♑, being measured upon your Line of Chords, will contain 20 degr. the *Sun's Declination*, he being in the 29. degree of ♒.

The like Declination *the* Sun *hath when he is in* 29 *degr. of* Taurus, *in* 1 *degr. of* Leo, *or* 29 *degr. of* Scorpio, *every of which Points are distant from one of the* Æquinoctial Points Aries *or* Libra 59 *degr.*

Prop. II.

The Latitude *of the Place, and the* Declination *of the Sun, being given, to find the* Ascensional Difference.

UPON the Projection this Proposition is to be resolved by finding the Side A B of the Right-angled Sphericall Triangle A ☉ B, Right-angled at B; which Triangle is compounded of three Arches of great Circles, namely, of A ☉, an Arch of the Horizon, A B, an Arch of the Æquinoctial, and ☉ B, an Arch of an Hour-Circle.

In this Triangle you have given (1.) the Side ☉ B, the Sun's Declination 20 d. (2.) the Angle ☉ A B, the Complement of the Latitude 38 d. 30 m. and the right Angle at B. In this Triangle therefore you have given ☉ B, the Perpendicular, and ☉ A B, the Angle at the Base, to find the Base A B, which you may doe by the 14. Case of Right-angled Sphericall Triangles. For which this is

The Analogie or Proportion.

As the Co-tangent of the Latitude 38 degr. 30 min. is to the Tangent of the Sun's Declination 20 degr.

So is the Radius 90 degr. to the Sine of the Ascensional Difference 27 degr. 14 min.

To resolve the Triangle upon the Projection,

Lay a Ruler to P, the Pole of the World, (and also of the Æquinoctial,) and the Point B, it will cut the Meridian Circle in the Point *b*; the distance *b* S, being taken in your Compasses and measured upon your Line of Chords, will reach from the beginning thereof to 27 degr. 14 min. the Ascensional Difference; which is so much as the Sun riseth or setteth before or after Six a Clock. So these 27 degr. 14 min. being turned into

into Time (by allowing 15 deg. for one Hour, and one Degree for 4 minutes of Time) is 1 Hour and 49 min. and so much doth the Sun rise or set before or after the hour of Six, according to the time or season of the Year: for if the Sun hath *North Declination*, then he *riseth before Six*, and *sets after*; but if the Sun have *South Declination*, then doth he *rise after*, and *set before* Six.

This Ascensional Difference *being added to 6 Hours will give you the* Semidiurnall Arch *or Half-length of the Day*; *and being taken from six Hours, will leave the* Seminocturnall Arch *or Half-length of the Night.*

The Semidiurnall Arch, *when the Sun hath* 20 *degr. of North Declination, is represented in the Projection by the Arch* ♊ *a* ☉, *and the* Seminocturnall Arch *by* ☉ ♌. *The* Semidiurnall Arch, *when the Sun hath* 20 *degr. of South Declination, is represented by the Arch* ♒ E r, *and the* Seminocturnall *by the Arch* r c ♐.

Though I have shewed how these may be found by adding *and* subtracting *the* Ascensionall Difference; *yet they may be found by the* Projection, *for the Arches are measured upon the* Æquinoctial. *Wherefore lay a Ruler to P, the Pole of the World, and the Point B, it will cut the Meridian Circle in* b: *So the distance* b Æ, *being measured by your Chord, will be* 117 *degr.* 14 *min. the* Semidiurnall Arch, *and* b æ *measured will be* 62 *degr.* 11 *min. for the* Seminocturnall Arch.

Prop. III.

The Latitude *of the Place, and the* Declination *of the Sun, being given, to find his* Amplitude.

THERE are two Triangles upon the Projection, by the resolving of either the Proposition may be resolved, and both of them Right-angled. The one is the Triangle P☉O,

Right-angled at O. The other is the Triangle made use of in the last Proposition, A ⊙ B. The first Triangle is constituted of these three Arches, viz. P O, an Arch of the Meridian, P ⊙, an Arch of an Hour-Circle, and ⊙ O, an Arch of the Horizon.———The second Triangle consisteth of A ⊙, an Arch of the Horizon, ⊙ B, an Arch of an Hour-Circle, and A B, an Arch of the Æquinoctial. In the first Triangle you have given the Perpendicular P O, the Latitude of the Place, and the Hypotenuse P ⊙, the Complement of the Sun's Declination 70 degr. by which you are to find the Base ⊙ O, the Sun's Amplitude from the North part of the Meridian, which may be found by the 7. Case of Right-angled Sphericall Triangles.—In the second Triangle B A ⊙ (which is that I will first exemplifie in) you have given (1.) the Perpendicular ⊙ B, the Sun's Declination 20 degr. (2.) the Angle at the Base ⊙ A B, the Complement of the Latitude 38 degr. 30 min. to find the Hypotenuse ⊙ A, the Sun's Amplitude from the East or West. So having the Perpendicular ⊙ B, and the Angle at the Base ⊙ A B, you may find the Hypotenuse ⊙ A by the 4. Case of Right-angled Sphericall Triangles. And for it this is

The Analogie or Proportion.

As the Co-sine of the Latitude 38 degr. 30 min. is to the Radius 90 degr.

So is the Sine of the Sun's Declination 20 degr. to the Sine of the Amplitude from the East or West Points of the Horizon 33 degr. 20 min.

To work the Proposition upon the Projection,

Lay a Ruler upon Z the Zenith, (which is also one of the Poles of the Horizon,) and to the Point ⊙, it will cut the Meridian Circle in *d*; and laid from Z to A, it will cut the Nadir Point in N: So the distance N *d*, being taken in your Compasses and measured upon your Line of Chords, will contain

tain 33 degr. 20 min. the quantity of the Hypotenuse, which is the Amplitude of the Sun's rising or setting from the true East or West Points of the Horizon.

To perform the same Work by the other Triangle P ⊙ O, Lay a Ruler to Z the Zenith, and the Point ⊙, and it will cut the Meridian Circle in the Point *d*, as before: So the distance *d* O, measured upon your Line of Chords, will contain 56 degr. 40 min. which is the Amplitude of the Sun's rising or setting from the North Point of the Horizon.

PROP. IV.

The Latitude *of the Place, and the* Declination *of the Sun, being given, to find the* Angle of the Sun's Position *at the time of his* rising.

THIS Proposition is resolvable upon the Triangle P ⊙ O: In which there is given (1.) the Hypotenuse P ⊙, the Complement of the Sun's Declination; (2.) the Perpendicular P O, the Latitude: and it is required to find the Angle P ⊙ O: which may be found by the 15. Case of Right-angled Sphericall Triangles, and by the following

Analogie or Proportion.

As the Co-sine of the Declination 70 degr. is to the Radius 90 degr.
So is the Sine of the Latitude 51 degr. 30 min. to the Sine of the Angle of the Sun's Position at the time of his rising.

By the Projection.

Lay a Ruler to X, (the Pole of the Hour-Circle P ⊙ S,) and the Point ⊙; the Ruler so laid will cut the Meridian Circle near the Point ♌: then set 90 degr. from ♌ to *x* upon the Meridian, and lay the Ruler from X to *x*; so shall it cut the Hour-

Circle S ☉ P (being continued without the Meridian Circle) in the Point ⊕. Again, the Ruler laid from ☉ to ⊕ will cut the Meridian Circle in the Point 2. So the distance O 2, being taken in the Compasses and applied to the Line of Chords, will be found to contain 56 degr. 29 min. which is the quantity of the Angle P ☉ O.

Prop. V.

The Sun's Declination, *and his* Amplitude *from the* North *part of the* Horizon, *being given, to find the* Latitude.

In the same Triangle as in the last Proposition P ☉ O, you have given (1.) the Base ☉ O, the Sun's Amplitude from the North part of the Horizon; (2.) the Hypotenuse P ☉, to find the Perpendicular P O. So there are two Sides given to find the third, which you may doe by the 8. Case of Right-angled Sphericall Triangles.

The Analogie or Proportion.

As the Co-sine of the Amplitude from the North 33 degr. 20 min. is to the Radius 90 degr.
So is the Sine of the Declination 20 degr. to the Co-sine of the Latitude 38 degr. 30 min.

By the Projection.

This is easie; for if you take in your Compasses the distance from O to P upon the Meridian, and measure it upon your Line of Chords, it will be found to contain 51 degr. 30 min. the Latitude required.

Prop.

PROP. VI.

The Sun's greatest Declination, *with his Distance from the next* Æquinoctial Point (Aries *or* Libra,) *being given, to find his* right Ascension.

THE Triangle A *k* ♒ in the Projection resolves this Proposition, in which you have given (1.) the Side A ♒, the distance of the ☉ from ♎; (2.) the Angle ♒ A *k*, the Sun's greatest Declination, and the right Angle at ♒: So you have given the Base, and the Angle at the Base to find the Hypotenuse, which you may doe by the Case of Right-angled Sphericall Triangles, and this

Analogie or Proportion.

As the Radius 90 degr. is to the Co-sine of the greatest Declination 66 degr. 30 min.
So is the Tangent of the Sun's distance from the next Æquinoctial point *Libra* 59 d. to the Tangent of the right Ascension 56 degr. 50 min.

By the Projection.

Lay a Ruler to P and *k*, it will cut the Meridian Circle in *s*: the distance S *s*, taken and measured upon your Line of Chords, will contain 56 degr. 50 min. the Sun's right Ascension.

I should here shew how the right Ascension and Declination of a Star might be found; but, the Calculation Trigonometricall being very laborious, I have therefore in this place omitted it, because I have in another Treatise, now speedily to be published, framed Tables of the Longitude, Latitude, Right Ascension, Declination,

clination, *and* Semidiurnall Arches, *of the most eminent Fixed Stars in the Heavens, exactly calculated for the Year of our Lord* 1670; *and also other Tables of the Sun's* right Ascension *for every Degree of the* Ecliptick *and Day of the Year: by help of which Tables, the Hour of the Night, the Rising, Setting, and the time of their coming to the South, may be obtained by the Rules and Directions prescribed in the forementioned Treatise, which will contain (besides the Tables here mentioned) all such others as at any time the Sea-man shall have occasion for in his Practice, and divers other things too tedious here to enumerate. But in the mean time I shall request the Reader to be satisfied with these two Tables following: These Exercises being things onely Practicall.*

A Table of the Sun's right Ascension for every Degree of the Ecliptick.

Degr.	♈		♉		♊		♋		♌		♍	
0	0	0	27	54	57	48	90	0	122	12	152	6
1	0	55	28	51	58	51	91	5	123	14	153	3
2	1	50	29	49	59	53	92	11	124	16	154	1
3	2	45	30	46	60	56	93	16	125	19	154	58
4	3	40	31	44	61	59	94	22	126	20	155	54
5	4	35	32	42	63	2	95	27	127	22	156	51
6	5	30	33	40	64	6	96	32	128	24	157	48
7	6	25	34	38	65	9	97	38	129	25	158	44
8	7	21	35	37	66	13	98	43	130	26	159	40
9	8	16	36	36	67	17	99	48	131	27	160	37
10	9	11	37	34	68	21	100	53	132	28	161	33
11	10	6	38	33	69	25	101	58	133	28	162	29
12	11	1	39	33	70	29	103	3	134	29	163	25
13	11	57	40	32	71	34	104	8	135	29	164	20
14	12	52	41	31	72	38	105	13	136	29	165	16
15	13	48	42	31	73	43	106	17	137	29	166	12
16	14	44	43	31	74	47	107	22	138	29	167	7
17	15	40	44	31	75	52	108	26	139	28	168	3
18	16	35	45	31	76	57	109	31	140	27	168	58
19	17	31	46	32	78	1	110	35	141	27	169	54
20	18	27	47	32	79	7	111	39	142	26	170	49
21	19	23	48	33	80	12	112	43	143	24	171	44
22	20	20	49	34	81	17	113	47	144	23	172	39
23	21	16	50	35	82	22	114	51	145	22	173	35
24	22	12	51	36	83	28	115	54	146	21	174	30
25	23	9	52	38	84	33	116	57	147	18	175	25
26	24	6	53	40	85	38	118	1	148	16	176	20
27	25	2	54	41	86	44	119	4	149	14	177	15
28	25	59	55	44	87	49	120	7	150	11	178	10
29	26	56	56	46	88	55	121	9	151	9	179	5
30	27	54	57	48	90	0	122	12	152	6	180	1

A Table of the Sun's right Ascension for every Degree of the Ecliptick.

Degr.	♎		♏		♐		♑		♒		♓	
0	180	0	207	54	237	48	270	0	302	12	332	6
1	180	55	208	51	238	51	271	5	303	14	333	3
2	181	50	209	49	239	53	272	11	304	16	334	1
3	182	45	210	46	240	56	273	16	305	19	334	58
4	183	40	211	44	241	59	274	22	306	20	335	54
5	184	35	212	42	243	2	275	27	307	22	336	51
6	185	30	213	40	244	6	276	32	308	24	337	48
7	186	25	214	38	245	9	277	38	309	25	338	44
8	187	21	215	37	246	13	278	43	310	26	339	40
9	188	16	216	36	247	17	279	48	311	27	340	37
10	189	11	217	34	248	21	280	53	312	28	341	33
11	190	6	218	33	249	25	281	58	313	28	342	29
12	191	2	219	33	250	29	283	3	314	29	343	25
13	191	57	220	32	251	34	285	8	315	29	344	20
14	192	52	221	31	252	38	286	13	316	29	345	16
15	193	48	222	31	253	43	287	17	317	29	346	12
16	194	44	223	31	254	47	288	22	318	29	347	7
17	195	40	224	31	255	52	289	26	319	28	348	3
18	196	35	225	31	256	57	290	31	320	27	348	58
19	197	31	226	32	258	1	291	35	321	27	349	54
20	198	27	227	32	259	7	292	39	322	26	350	49
21	199	23	228	33	260	12	293	43	323	24	351	44
22	200	20	229	34	261	17	294	47	324	23	352	39
23	201	16	230	35	262	22	295	51	325	22	353	35
24	202	12	231	36	263	28	296	54	326	21	354	30
25	203	9	232	38	264	33	297	57	327	18	355	25
26	204	6	233	40	265	38	298	1	328	16	356	20
27	205	2	234	41	266	44	299	4	329	14	357	15
28	205	59	235	44	267	49	300	7	330	11	358	10
29	206	56	236	46	268	55	301	9	331	9	359	5
30	207	54	237	48	270	0	302	12	332	6	360	0

The right Aſcenſion, Declination, and Magnitude of ſome principal Fixed Stars.

The Stars Names.	Right Aſcenſion.		Declination.			Magnitude.
	D.	M.	D.	M.		
The Pole-Star	7	47	87	27	N	2
The Girdle of *Andromeda*	12	32	33	48	N	2
The former Horn of the Ram	23	38	17	37	N	4
Bright Star in the Ram's Head	26	56	21	48	N	3
The Whale's Jaw	41	3	2	42	N	2
The Head of *Meduſa*	41	27	39	35	N	3
The Bull's Eye	64	0	15	46	N	1
The Goat	72	44	45	35	N	1
The former Shoulder of *Orion*	76	38	4	59	N	2
The latter Shoulder of *Orion*	84	7	7	18	N	2
The great Dog	97	27	16	13	S	1
The uppermoſt Head of the Twins	108	1	32	35	N	2
The little Dog	110	17	6	6	N	2
The lower Head of the Twins	111	0	28	49	N	2
The Crib	125	4	20	52	N	Neb.
Hydra's Heart	137	39	7	10	S	2
Lion's Heart	147	27	13	39	N	1
Lion's Loins	163	54	22	26	N	2
Lion's Tail	172	49	16	32	N	1
The Virgin's Girdle	189	32	5	20	N	3

The right Ascension, Declination, and Magnitude of some principal Fixed Stars.

The Stars Names.	Right Ascension.		Declination.			Magnitude.
	D.	M.	D.	M.		
Aliot	189	36	57	36	N	2
Vindemiatrix	191	15	12	51	N	3
The Virgin's Spike	196	44	9	17	S	1
Arcturus	209	56	21	4	N	1
The Southern Balance	217	56	14	32	S	2
The Northern Balance	224	31	8	2	S	2
In the Serpent's Neck	231	49	7	35	N	3
The Scorpion's Heart	242	4	25	34	S	1
Hercules Head	254	40	14	51	N	3
Ophiuchus Head	259	41	12	52	N	3
The Harp	276	17	38	30	N	1
The Vulture	293	27	8	1	N	2
The upper Horn of the Goat	299	30	13	32	S	3
Left Hand of *Aquarius*	307	10	10	43	S	4
Left Shoulder of *Aquarius*	318	18	7	2	S	3
Pegasus Mouth	321	49	8	18	N	3
Right Shoulder of *Aquarius*	326	59	1	58	S	3
Fomahant	339	29	31	23	S	1
Upper Wing of *Pegasus*	341	53	13	21	N	2
In the tip of *Pegasus* Wing	358	52	13	15	N	2

PROP.

Prop. VII.

The Latitude *of the Place and the Sun*'s Declination *being given, to find at what Hour the Sun will be upon the true* East *or* West *Points.*

UPON the Projection there are two Right-angled Sphericall Triangles, by either of which this Proposition may be solved. The one is the Triangle Z P *o*, made by the Intersections of Z *o*, an Arch of the Prime Verticall, P *o*, an Arch of an Hour-Circle, and Z P, an Arch of the Meridian. In which Triangle there is given Z P, the Perpendicular, the Complement of the Latitude of the Place 38 degr. 30 min. and the Hypotenuse P *o*, the Complement of the Sun's Declination 70 degr. to find the Angle at the Perpendicular Z P *o*, which you may doe by the 14. Case of Right-angled Sphericall Triangles.

The other Triangle is *o* C A, right-angled at C, and is constituted of *o* C, an Arch of an Hour-Circle, C A, an Arch of the Æquinoctial, and *o* A, an Arch of the Prime Verticall. In which Triangle you have given, (1.) the Perpendicular, O C, the Sun's Declination; (2.) the Angle at the Base, C A *o*, the Latitude 51 degr. 30 min. to find the Base C A. Thus having the Perpendicular and the Angle at the Base, you may find the Base C A as followeth, this being

The Analogie or Proportion.

As the Tangent of the Latitude 51 degr. 30 min. is to the Tangent of the Sun's Declination 20 degr.

So is the Radius 90 degr. to the Co-sine of the Hour from Noon.

To resolve the Proposition by the Projection,

Lay a Ruler upon P, the Pole of the World, and the Angle C of your Triangle, the Ruler will cut the Meridian Circle in the Point *g*: So *g* Æ, being taken in your Compasses and measured upon your Line of Chords, will be found to contain 73 degr. 10 min. which converted into Hours and Minutes will be 4 hours and about 53 min. So that the Sun, when he hath 20 degr. of Declination, will come to the East Point at 7 min. past 7 in the Morning, and will be due West 53 min. after 4 in the Afternoon.

Prop. VIII.

Having the Latitude *of the Place, and the Sun's* Declination, *given, to find what* Altitude *the Sun shall have when he is upon the true* East *or* West *Points.*

THIS Proposition may be resolved by either or both of the Triangles mentioned in the last Proposition. For in the Triangle Z P *o* you have given P Z, the Perpendicular, and P O, the Hypotenuse, to find Z *o*, the Base, by the 8. Case of Right-angled Sphericall Triangles.

But in the Triangle *o* C A, you have given (1.) the Perpendicular *o* C, the Sun's Declination 20 degr. (2.) the Angle at the Base *o* A C, the Latitude of the Place 51 degr. 30 m. to find the Hypotenuse *o* A, for which this is

The Analogie or Proportion.

As the Sine of the Latitude 51 degr. 30 min. is to the Sine of the Declination 20 degr.

So is the Radius 90 degr. to the Sine of the Sun's Altitude being due East or West 25 degr. 55 min.

To resolve the Proposition by the Projection,

Lay a Ruler upon O, (one of the Poles of the Prime Verticall) and to the Angle *o* of the Triangle; a Ruler thus laid will cut the Meridian Circle in the Point *p*: So the distance H *p*, being taken and measured upon the Line of Chords, will be 25 degr. 55 min. and such height will the Sun have when he is either East or West.

Prop. IX.

The Latitude *of the Place, and the* Declination *of the Sun, being given, to find what* Altitude *the Sun shall have at* Six *of the Clock.*

FOR finding of the Triangles upon the Projection, which will resolve this and the following Propositions, you must suppose another Azimuth Circle to be drawn in the Projection from Z to N, and through that Point where the Parallel of Declination ♊☉♌, and the Axis of the World, or Hour-Circle of Six, P A S, do cross each other. The drawing of which Azimuth Circle I purposely omitted, chiefly because the Scheme in that place is more cumbred with Lines and Letters then any other part thereof: But you may well enough, for the solving of these two Propositions, imagine it to be drawn, the Pole whereof is at *. This Azimuth Circle being supposed to be drawn, you have upon the Projection two Triangles like-angled, which will perform the Work of resolving this Proposition. In one of which you have given the Base, which is the Complement of the Declination, and the Perpendicular, which is the Complement of the Latitude, to find the Hypotenuse, which is the Complement of the Sun's Altitude required. This Triangle may be resolved by the first Case aforegoing.———In the other Triangle there will be given the

Hypotenuse,

Hypotenuſe, which is the Sun's Declination, and the Angle at the Baſe, which is the Latitude, to find the Perpendicular, which is the Sun's Altitude at Six a Clock: To find which this is

The Analogie or Proportion.

As the Radius 90 degr. is to the Sine of the Sun's Declination 20 degr.
So is the Sine of the Latitude 51 degr. 30 min. to the Sine of the Sun's Altitude at Six 15 degr. 30 min.

To reſolve the Propoſition by the Projection,

Lay a Ruler upon the Point *, and that Point where the Parallel of Declination ♊☉♌ croſſeth the Axis or Hour of Six; the Ruler thus laid will cut the Meridian Circle in the Point *g*. So O *g*, being meaſured upon the Chords, will give you 15 degr. 30 min. And ſuch Altitude will the Sun have at the Hour of Six in the Latitude of 51 degr. 30 min. when he hath 20 degr. of Declination.

Prop. X.

The Latitude *of the Place and the* Declination *of the Sun being given, to find what* Azimuth *the Sun ſhall have at* Six *a Clock.*

THE two Triangles that were ſuppoſed in the laſt Propoſition to be drawn upon the Projection will reſolve this Propoſition alſo; but ſeeing the Triangles are not drawn, but ſuppoſed, I will onely give you the Analogie, and then the way of working it upon the Projection.

The Analogie or Proportion.

As the Co-ſine of the Latitude 38 degr. 30 min. is to the Radius 90 degr.

So

So is the Co-tangent of the Sun's Declination 70 degr. to the Tangent of the Sun's Azimuth counted from the North part of the Meridian 77 degr. 14 min.

To resolve the Proposition upon the Projection,

Lay a Ruler to the Zenith-point Z, and upon the Point where the Parallel of Declination cuts the Hour-Circle of Six; the Ruler thus laid will cut the Meridian Circle in *r*: So the distance *r* O, being measured upon the Line of Chords, will contain 77 degr. 14 min. which is the Azimuth from the North part of the Meridian.——The distance N *r*, measured upon the Chords, will give you 12 degr. 46 min. which is the Azimuth from East or West.——And *r* H, measured upon your Chord, will contain 102 degr. 46 min. his Azimuth from the South.

Prop. XI.

The Latitude *of the Place, the* Declination *of the Sun, and the Sun's* Altitude, *being given, to find the Sun's* Azimuth *either from the* East, North *or* South *Points of the* Horizon.

ALL the foregoing Propositions have been performed by the resolving of a Right-angled Sphericall Triangle: This and some others following require the resolving of an Oblique-angled Triangle for their Solution. So this Proposition is performed upon the Oblique-angled Triangle Z E P, which in the Projection is constituted by the Intersection of P E S, an Hour-Circle, Z E N, an Azimuth Circle, and Z H N O, the Meridian: and the Arches of these Circles intersecting each other in the Points Z, E, and P, do make the Triangle Z E P; in which you have given the three Sides, (1.) Z P, the Complement of the Latitude 38 degr.

30 min. of the Place, (2.) P E, the Complement of the Sun's Declination South 110 degr. (3.) Z E, the Complement of the Sun's Altitude 78 d. to find the Angle E Z P, which may be resolved by the 11. Case of Oblique-angled Sphericall Triangles. But this being to resolve a particular Proposition, I shall give another Analogie or Proportion whereby to work it by the Canon.

Adde all the three Sides together into one Sum, and take the half thereof, from which half Sum subtract the Side P E, noting the difference, as here you see done.

		d.	m.
The Side	Z P	38	30
	P E	110	00
	Z E	78	00
The Sum		226	30
The half Sum		113	15
The difference between the half Sum and P E		3	15

Having found the Sum, the half Sum, and the difference, you may work by the following

Analogie or Proportion.

(1.) As the Radius 90 degr. is to the Co-sine of the Altitude 78 degr.
So is the Co-sine of the Latitude 38 degr. 30 min. to the Sine of a fourth number, which is 37 degr. 30 min.
(2.) As the Sine of the fourth number 37 degr. 30 min. is to the Sine of the half Sum 113 degr. 15 min.
So is the Sine of the difference 3 degr. 15 min. to another Sine, *viz.* 4 degr. 54 min. Unto which seventh Sine if you adde the Sine of 90 degr. half that Sum shall be the Sine of an Arch, whose Complement being doubled is the Azimuth from the North.

To

To resolve this Probleme by the Projection.

That which is most intricate and difficult to perform by Numbers, is by Projection effected with the same ease as any of the rest. As in this Proposition, it is the Angle E Z P which is required.———Lay a Ruler upon the Zenith-point Z, and to the Point G, upon the Horizon ; the Ruler thus laid will cut the Meridian Circle in the Point *g*. So the distance *g* O, being taken in your Compasses and measured upon your Line of Chords, will be found to contain 146 degr. which is the Sun's Azimuth from O, the North part of the Meridian.———But if you measure the distance between the *g* and H, it will contain 34 degr. which is the Azimuth from H, the South part of the Meridian.———And if you measure the distance *g* N upon your Chord-Line, you shall find that to contain 56 degr. and so much is the Sun's Azimuth from A, the East and West Points of the Horizon.

This Example of finding the Azimuth was taken when the Sun had 20 *degr. of South Declination. I will now farther exemplifie this Proposition by finding the Azimuth when the Sun hath North Declination.——As let the Latitude be as before* 51 *d.* 30 *min. the Sun's Altitude* 12 *degr. and the Declination* 20 *d. North.*

To work this by the Canon of Sines differeth nothing from the former, for the Analogie or Proportion is general in all Cases.

Upon the Projection it is resolved (though the same way, yet) upon another Triangle, namely, the Triangle Z P *a*, in which is given (1.) Z P, the Complement of the Latitude 38 d. 30 min. (2.) Z *a*, the Complement of the Altitude 78 degr. (3.) the Complement of the Sun's Declination North 70 degr. and you are to find the Angle P Z *a*, the Sun's Azimuth from the North.

Lay a Ruler upon Z unto the Point *a*, it will cut the Meridian Circle in the Point *s*; the distance *s* O, being taken in your Compasses and applied to your Line of Chords, will there give you 72 degr. 52 m. And such is the Sun's Azimuth from the North.

If you subtract this Azimuth from the North 72 *degr*. 52 *m. from* 180 *degr. the remainer* 107 *degr.* 8 *min. will give you the Azimuth from the South, which upon the Projection is the distance* s H.——*And if from this Azimuth from the South* 107 *d.* 8 *min. you take* 90 *degr. the remainer* 17 *degr.* 8 *min. is the Azimuth from the East or West, which in the Projection is the distance* N s.

Prop. XII.

The Latitude *of the Place, the Sun's* Declination, *and the Sun's* Altitude, *being given, to find the* Hour *of the* Day.

THIS Proposition is performed by the resolving of the Oblique-angled Sphericall Triangle Z P *a*, composed of Z P, an Arch of the Meridian, Z *a*, an Arch of an Azimuth Circle, and of P *a*, the Arch of an Hour-Circle: In which you have given (as in the last Proposition) the three Sides, to find the Angle Z P *a*, which you may doe by the 11. Case of Oblique Sphericall Triangles.

To resolve this Proposition by the Canon; Adde the three Sides together, and from the half Sum of them subtract the Complement of the Sun's Altitude, and note the difference, as you see here done.

The

		d.	m.
The Side	Z P, the Complement of the Latitude	38	30
	Z *a*, the Complement of the Altitude	78	00
	P *a*, the Complement of the Declination	70	00
	The Sum	186	30
	The half Sum	93	15
	The difference between the half Sum and Z *a*, the Complement of the Altitude	15	15

Being thus prepared, you may resolve the Proposition by the Canon of Sines, by this

Analogie or Proportion.

(1.) As the Radius 90 degr. is to the Co-sine of the Sun's Altitude 78 degr.
So is the Co-sine of the Latitude 38 degr. 30 min. to a fourth Sine, viz. 35 degr. 48 min.
(2.) As this fourth Sine of 35 degr. 48 min. is to the Sine of the half Sum 93 degr. 15 min.
So is the Sine of the Difference 15 degr. 15 min. to another Sine, viz. to the Sine of 26 degr. 40 min. Unto which Sine if you adde the Sine of 90 degr. (or Radius,) half that Sum shall be the Sine of an Arch, whose Complement being doubled is the Hour from the Meridian 95 degr. 52 min.

To resolve the Proposition by the Projection.

In the Triangle Z P *a*, it is the Angle at P that is to be found. Wherefore lay a Ruler from the Point P to the Point *a*, and it will cut the Meridian Circle in *t*: So the Arch *t* Æ, being measured upon your Line of Chords, will be found to contain 95 d. 52 min. which is the Hour from the Meridian; and the Arch *t a*, being measured, will contain 84 degr. 8 min. which is the

Hour from Midnight. Also the Arch *t* S, being measured upon the Chord, will contain 5 degr. 52 min. the Hour from Six.

		d.	m.		hours	m.
The Arch	*t* Æ	95	52	converted into Time is	6	23
	t a	84	08		5	36
	t S	05	52		0	23

To convert Degrees and Minutes of the Æquinoctial into Hours and Minutes of Time: Note that 15 *Degrees of the Æquinoctial make one Hour of Time, and one Degree* 4 *Minutes of Time. Therefore divide the Degrees of the Æquinoctial by* 15, *the Quotient is Hours; and multiply the Degrees by* 4, *and the Product will be Minutes of Time.*——*So the Hour from the Meridian being* 95 *degr.* 52 *min. divide* 95 *by* 15, *the Quotient is* 6 *Hours, and* 5 *remaining, which* 5 *multiply by* 4, *and it makes* 20 *Minutes of Time, and the* 52 *min. make* 3 *minutes of Time and more, almost* 4 *minutes. So that* 95 *degr.* 52 *min. of the Æquinoctial do make in Time* 6 *hours and almost* 24 *minutes.*

PROP. XIII.

The Declination, Altitude, *and* Azimuth *of the Sun, being given, to find the* Hour *of the Day.*

THE Triangle Z E P in the Projection resolves this Problem: in which there is given (1.) E P, the Complement of the Sun's Declination 70 degr. South; (2.) the Side E Z, the Complement of the Sun's Altitude 78 degr. and (3.) the Angle E Z P, the Sun's Azimuth from the North. So that you have two Sides and an Angle opposite to one of them given, to find the Angle opposite to the other, which you may doe by the 8. Case of Oblique-angled Sphericall Triangles, and by the following

Ana-

Analogie or Proportion.

As the Co-sine of the Declination 70 degr. is to the Sine of the Azimuth 146 degr. or 34 degr.
So is the Co-sine of the Altitude 78 degr. to the Sine of the Hour from Noon 35 degr. 36 min.

By the Projection.

Lay a Ruler to the Pole P, and upon the Point D in the Æquinoctial; a Ruler thus laid will cut the Meridian Circle in *e*; the distance *e* Æ, being measured upon the Line of Chords, will give you 35 degr. 36 min. the Hour from Noon, which in Time is 2 hours and 22 minutes.

Prop. XIV.

The Sun's Declination, *his* Altitude, *and the* Hour *from Noon, being given, to find the Sun's* Azimuth *from the North part of the Meridian.*

In the same Triangle Z E P you have given (1.) the Side E P, the Complement of the Sun's Declination South 70 d. (2.) the Side Z E, the Complement of the Altitude 78 degr. and (3.) the Angle Z P E, the Hour from Noon 36 degr. 35 m. That is, you have given (as before) two Sides and an Angle opposite to one of them, to find the Angle opposite to the other, which you may doe by the 8. Case of Oblique Sphericall Triangles; or by this

Analogie or Proportion.

As the Co-sine of the Altitude 78 degr. is to the Sine of the Hour from Noon 36 degr. 35 min.
So is the Co-sine of the Sun's Declination 70 d. to the Sine of the Azimuth from the North part of the Meridian 146 degr. or 34 degr. from the South.

By

By the Projection.

Lay a Ruler to Z, and upon the Point G, which will cut the Meridian Circle in *g*: So *g* H, being meaſured upon your Line of Chords, will be found to contain 34 degr. the Azimuth from the South part of the Meridian, which being taken from 180 degr. the remainer will be 146 degr. equal to the Arch *g* O, the Azimuth from the North part of the Meridian.

Prop. XV.

The Hour *from Noon, the* Latitude *of the Place, and the* Altitude *of the Sun, being given, to find the* Angle *of the* Sun's Poſition.

IN the Oblique-angled Sphericall Triangle Z P E you have given the Side Z P, the Latitude, Z E, the Complement of the Sun's Altitude, and Z P E, the Hour from Noon, to find the Angle Z E P, which is the Angle of the Sun's Poſition at the time of the Queſtion: So in the Triangle Z P E you have two Sides, with an Angle oppoſite to one of them, given, to find the Angle oppoſite to the other Side, which you may find by the 2. Caſe of Oblique-angled Sphericall Triangles: For which this is

The Analogie or Proportion.

As the Co-ſine of the Sun's Altitude 78 degr. is to the Sine of the Hour from Noon 35 degr. 36 min.
So is the Co-ſine of the Latitude 38 degr. 30 min. to the Sine of the Angle of the Sun's Poſition at the time of the Queſtion 21 degr. 45 min.

By the Projection.

This is the moſt troubleſome Propoſition that we have yet met

met withall to be resolved by the Projection; and yet it is also thereby easily resolved in this manner.

Take in your Compasses 90 degr. of your Chords; then lay a Ruler upon Y, (the Pole of the Hour-Circle P E S,) and the angular Point E; it being so laid will cut the Meridian Circle in *v*. Then set 90 degr. of your Line of Chords from *v* to *x* upon the Meridian Circle, and the Ruler laid from Y to *x* will cut the Hour-Circle P E S in the Point *y*.

Again, lay a Ruler to ⊙, (the Pole of the Azimuth Circle Z E N,) and to the angular Point E; it being so laid will cut the Meridian Circle in the Point M. Set 90 degr. from M to *z* upon the Meridian Circle, and lay a Ruler upon ⊙ and *z*; it will cut the Azimuth Circle (it being continued without the Meridian Circle) in the Point δ.

Lastly, Lay a Ruler to the angular Point E, and this Point δ, it will cut the Meridian Circle in ϑ; also lay a Ruler from E to *y*, it will cut the Meridian in λ. The distance $\theta\ \lambda$, being taken in the Compasses and measured upon your Line of Chords, will contain 21 degr. 45 min. and that is the quantity of the enquired Angle Z E P, which is the Angle of the Sun's Position at the time of the Question.

Prop. XVI.

The Sun's Altitude, *his* Declination, *and* Azimuth *from the* North, *being given, to find the* Latitude.

IN the former Triangle Z E P, let there be given (1.) E Z, the Complement of the Sun's Altitude 78 degr. (2.) E P, the Complement of the Sun's Declination (or his distance from the North-pole) 110 degr. (3.) the Angle E Z P, the Azimuth from the North.———By the last Proposition find the Angle of the Sun's Position, Z E P, which had, the Side Z E, the Complement of the Latitude, may be found by this

Analogie or Proportion.

As the Sine of the Sun's Azimuth 146 degr. (or 34 degr.) is to the Sine of the Sun's distance from the North-pole 110 d. (or 70 degr.)

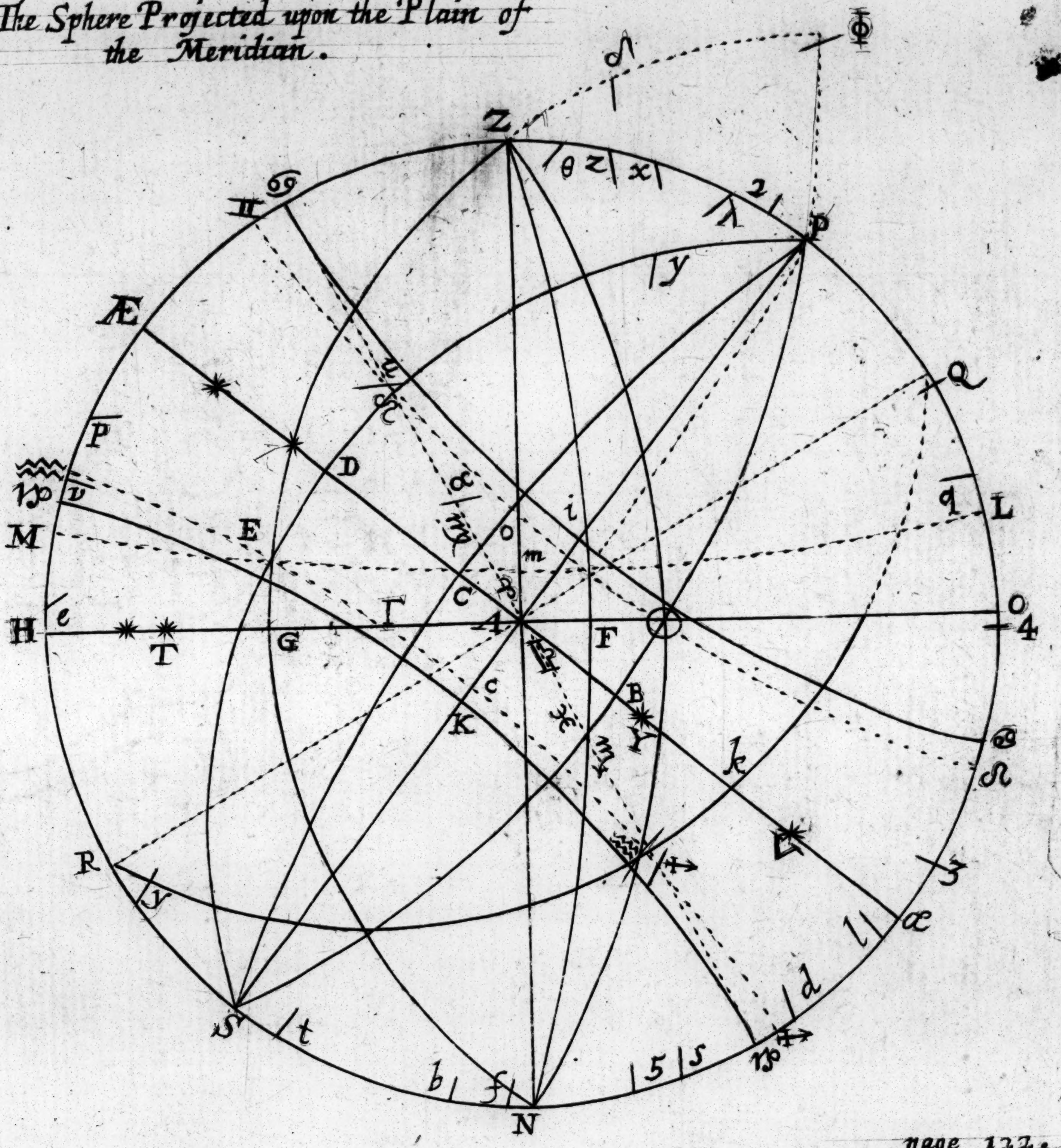
So is the Sine of the Angle of the Sun's Position 21 degr. 45 m. to the Complement of the Latitude 38 degr. 30 min.

By the Projection.

It being the Side Z P that is required, forasmuch as that is an Arch of the Meridian Circle, you have no more to doe but to take the distance Z P in your Compasses, and measure it upon your Line of Chords; which being done, you will find it will contain 38 degr. 30 m. the Complement of the Latitude, which taken from 90 degr. leaves 51 degr. 30 m. the Latitude it self.

By the Solution of the foregoing Propositions, both by Trigonometricall Calculation by the Canons of Sines and Tangents, and also by the Projecting of the Sphere, you may easily discern the facility of the one above the other, namely, that of Projection; forasmuch as those Propositions which by Calculation are most troublesome, are by the Projective way most easie.——For, here is no need of letting fall Perpendiculars to reduce the Oblique Triangle into two Right-angled Triangles, and so making of two works for solving of one Problem. Besides, the Projection renders every Triangle so naturally to the eye, that they are resolved (as it were) by looking on them.———Many more Examples I might have given upon this one Projection, but I see the Scheme grow too full of Lines and Letters, that makes me here break off. And now I will shew how the ingenious young Seaman may apply these Propositions to his use at Sea.

The Sphere Projected upon the Plain of the Meridian.



The foregoing PROPOSITIONS applied to Practice:

By which the Ingenious young Sea-man may make them serviceable to him at Sea, to severall good and usefull Purposes.

The Seventh EXERCISE.

OF *the foregoing* Astronomicall Propositions *the ingenious* Sea-man *may make good and profitable use at Sea. For some of them will be assistent to him in the finding of the* Latitude *of the Place he is in.——Some of them will help him to find* the time of the Sun's rising and setting *in any place, and at any time of the Year.——Some will help him to the* Hour *of the* Day,——*Some to the* Hour *of the* Night, *at any time and in any place.——And divers of them to find the* Variation of the Compass. *Examples of all which I will instance in, so that he may put them in practice at Sea.*

I. *Propositions assistent to find the Latitude.*

THE *Propositions* which may be applied to the finding of the *Latitude* are the First and the Sixteenth.

The first *Proposition* is to find *The Sun's Declination*, which being obtained, and the Sun's *Meridian Altitude* observed at *Sea* or *Land* in any part of the World, the *Latitude* of that Place, by help of them, may be known; in which there are severall *Cases*, according as the Sun hath either *North* or *South Declination*, and as the *Sun* is situate, he being either upon the *North* or *South-side* of the *Meridian*.——The severall Varieties are these which follow.

When the Sun is in the *Æquinoctial*, having no *Declination*, and the *Meridian Altitude* is observed on the

- SOUTH-side of the *Meridian*, — The *Meridian Altitude* taken from 90 degr. leaves the Elevation of the *North-pole*.
- NORTH-side of the *Meridian*, — The *Meridian Altitude* taken from 90 degr. leaves the Elevation of the *South-pole*.

When the Sun's *Declination* is

- NORTH, — If the *Meridian Altitude* be less then 90 d. and the Sun upon the *South-side* of the *Meridian*; the *Sun's Declination*, being taken from the *Meridian Altitude*, leaves the height of the *Æquinoctial*, which taken from 90 d. gives the *Latitude North*.
- SOUTH, — If the *Meridian Altitude* be less then 90 d. and the Sun upon the *South-side* of the *Meridian*, adde the *Meridian Altitude* and *Declination* together; their Sum is the height of the *Æquinoctial*, which taken from 90 degr. leaves the *Latitude North*. But if the Sum of the *Declination* and *Altitude* exceed 90 degr. take 90 degr. therefrom, the remainer is the *Latitude South*.

When

When the Sun's *Declination* is NORTH, If the *Meridian Altitude* be leſs then 90 d. and the Sun upon the *North-ſide* of the *Meridian*, adde the *Altitude* and *Declination* together; their Sum is the height of the *Æquinoctial*, which taken from 90 d. leaves the *Latitude South.*—But if the Sum be above 90 d. take 90 d. therefrom, the remainer is the *Latitude North.*

SOUTH, If the *Meridian Altitude* be leſs then 90 d. and the Sun upon the *North-ſide* of the *Meridian*, ſubtract the *Declination* from the *Meridian Altitude*; the remainer is the height of the *Æquinoctial*, which taken from 90 degr. leaves the *Latitude South.*

When the Sun's *Declination* is { NORTH, / SOUTH, } If the *Meridian Altitude* be juſt 90 degr. the *Sun's Declination* is the *Latitude* { *North.* / *South.* }

If the *Meridian Altitude* be obſerved under the *Pole*, within the bounds of the *Polar Circles*, in ſuch caſe the *Sun's Declination* muſt be taken from 90 degr. and what remains is his diſtance from the *Pole*; which being added to the *Meridian Altitude*, the Sum is the *Latitude* of the Place.

The other Propoſition which will be aſſiſtent to find the *Latitude* is the Sixteenth. If you ſet your *Compaſs* to the given *Azimuth*, and when the Sun is upon that *Azimuth*, if you take his height, you may find your *Latitude* either by *Trigonometricall Calculation* or by *Projection*. As in the Sixteenth Propoſition.

II. *Propositions assistent to find the time of the Sun's* rising *and* setting.

THE principal Proposition for this purpose is the Second, which is to find the *Ascensionall Difference*, from which the time of the *Sun's rising* and *setting*, the *Semidiurnall* and *Seminocturnall Arches*, may be gathered; and from thence the length of the *Day* and *Night*: all which are plainly shewed in the Proposition it self.

III. *Propositions assistent to find the* Hour *of the* Day *and* Night.

THE Twefth and Thirteenth Propositions will be serviceable to find the Hour of the Day. The Twelfth giving the Hour at any time of the Day by the Work of that Proposition it self.——The Thirteenth findeth the *Hour* upon a given *Azimuth* and *Altitude*. Wherefore set your Compass to the given *Azimuth*, and observe his *Altitude* when he cometh upon that *Azimuth*; the Sun's *Declination* (or time of the year) being known, you may then find the *Hour* by the Work of the Thirteenth Proposition.

The Proposition that will be assistent to you in finding the *Hour* of the *Night* is chiefly the Sixth, it shewing how to find the Sun's *right Ascension*, which, with the assistence of those other Tables which follow that *Proposition*, will help you to the *Hour* of the *Night*, and also to find at what time any of the *Stars* there inserted in the *Table* will be upon the *Meridian*. The manner how to effect either shall be shewed in these two following Problems.

PROBL.

PROBL. I.

How to find at what time any of the Stars *in the Table of the Sixth* Proposition *will be upon the* Meridian.

SUbstract the *Right Ascension* of the Sun from the *Right Ascension* of the *Star*, the remainer is the time of the *Star's* coming to the *Meridian* after noon. But if the *right Ascension* of the *Star* be less then the *Right Ascension* of the *Sun*, adde 360 degr. thereto, and substract the *Right Ascension* of the *Sun* from the Sum, and the Remainer is the time of the *Star's* coming to the *Meridian*.

Example. Upon the fourth of *October* 1667, the Sun being in 21. degr. of *Libra*, I would know at what time *Sirius* (or the *Great Dog*) will be upon the *Meridian*.

	d.	m.
The *Right Ascension* of *Sirius* is	97	27
The *Right Ascension* of the *Sun* that day is	199	23
Because Substraction cannot be made, adde 360 d. to the *Right Ascension* of the *Star*	360	00
The Sum is	457	27
The *Sun's Right Ascension* substracted from 458 degr. 4 min. leaves the time of the *Star's* coming to the *Meridian*	258	04

Which 258 degr. 4 min. being converted into Time make 17 hours almost, that is, at 5 of the Clock the next Morning *Sirius* will be upon the *Meridian*.

Probl. II.

To find the Hour *of the* Night *by any of the* Stars *that are in the* Table *of the Sixth* Proposition.

TAKE the *Altitude* of the *Star*, and by his *Declination* and *Altitude* find the Hour (by the Twelfth Proposition) as if it were by the Sun, which I call the *Star's Hour*. Then comparing the *Right Ascension* of the *Sun* with the *Right Ascension* of the *Star*, you may come to find the *Hour* of the *Night*.

Example. Upon the 16. day of *November*, in the Morning, I took the *Altitude* of *Arcturus*, finding it to be 27 degr. 12 m. and his *Declination* (by the Table) I find to be 20 degr. 58 m. By help of these two and the *Latitude* I find the *Star's Hour* to be 72 degr. Then compare the *Sun's Right Ascension* with the *Star's Right Ascension*, and find his time of coming to the *Meridian*, as in the former *Probl.* the difference between the *Star's Hour* and his coming to the *Meridian* is the *Hour* of the *Night*. See the manner of the Operation.

	d.	m.
The *Right Ascension* of *Arcturus*	210	13
The *Right Ascension* of the *Sun*	242	00
Adde to make Subt.	360	00
The Sum is	570	13
The *Sun's Right Ascension* substracted, rests	328	13
From which take 180 degr. or 12 hours	180	00
Rests	148	13
The *Star's Hour* substracted	72	00
Leaves the *Hour* of the *Night*	76	13

Which converted into Time is 5 h. 5 m. and that is the Hour in the Morning.

IV *Pro-*

IV. *Propositions assistent to the finding of the* Variation of the Compass.

THE Propositions that will be serviceable herein are the 3.7.8.9. 10. 11. and 14. but more especially the *Third* and the *Eleventh:* and those I shall here illustrate by *Example*, though all the rest (as occasion may fall out) will be also usefull thereunto. By the Third Proposition you may find the Amplitude of the Sun's Rising and Setting.——By the Eleventh you may find the Azimuth at any time of the day.— By either of which the *Variation of the Compass* may be found, and also which way it varieth.

I. *To find the Variation by the* Amplitude.

BY the Third Proposition you found the Sun's *Amplitude* at his *rising* or *setting* to be 33 degr. 20 min. from the true *East* or *West* Points of the *Horizon* towards the *North*. Having thus before-hand found the *Amplitude*, in the Morning I set my *Compass* to the Sun at his Rising; and if I find that the Sun by my Compass do rise 33 d. 20 m. from the *West*-point thereof towards the *North*, then may I be ascertain'd that my *Compass* hath no *Variation*, but that the *Fly* or *Wires* do point directly *North* and *South*.——But finding before-hand the *Amplitude* to be 33 d. 20 m. and I should find the Sun to rise but 28 degr. from the *East*-point of my Compass, then substracting 28 degr. from 33 degr. 20 min. the difference is 5 degr. 20 min. and so much doth my *Compass* varie from the true *East*-point, and consequently all the other Points of the *Compass* as much.

Now to find which way the *Compass* varieth, you must observe whether your *Amplitude*, found by your *Calculation*, be to the *Right* or *Left-hand* of the *Sun's rising* or *setting*. And

S if

if it be on the *Right-hand*, you may conclude the *Variation* to be *Easterly*; but if on the *Left-hand*, *Westerly*.

As for *Example*; Finding by the *Amplitude* that the Sun should rise 33 d. 20 min. from the *East Northerly*, when I come to set my *Compass* to the Sun at his *rising*, I find that the Sun riseth but 28 degr. from the *East Northerly*; wherefore the *Amplitude* found is on the *Left-hand*, and so I conclude the *Variation* to be 5 d. 20 min. *Westerly*.

II. *To find the Variation by the* Azimuth.

SUppose the Sun's *Azimuth* found by the Eleventh *Proposition* to be 107 degr. 30 min. from the *North*, and when I set the *Compass*, I find the *Magneticall Azimuth* to be 102, the difference between the *true* and the *Magneticall Azimuth* being 5 d. 30 m. which is the *Variation*.

Now to know whether this *Variation* be towards the *East* or towards the *West*: seeing by the *Azimuth* found the Sun should have been 107 d. 30 min. from the *North*, which is 17 degr. 30 min. from the *East*; but setting of the *Sun* with my *Compass*, I find that it was from the *East* to the *Southward* onely 12 degr. so that the Degree upon which the Sun should have been was more towards the *Right-hand* then the Degree on which it was; therefore I conclude the *Variation* to be 5 degr. 30 min. *Easterly*.

PRO-

PROPOSITIONS GEOGRAPHICALL,

Shewing how the Distance of any two Places upon the Terrestriall Globe may be found, both by Trigonometricall Calculation and Geometricall Projection.

The Eighth EXERCISE.

B*Efore I come to shew how the* Distances *of* Places *are to be found upon the* Terrestriall Globe, *it will be necessary, first, to describe unto you the manner how any two* Places, *whose* Longitudes *and* Latitudes *are given, may be laid down upon the* Projection. *Wherefore in the* Scheme, *the outward Circle thereof, N Æ S æ, represents the* first Meridian, *passing over the* Islands *of* Azores, *from whence the* Ancients *did begin their account of* Longitude, *because (say some) the* Compass *hath there no* Variation.——*The Line Æ æ is the* Æquinoctial, *upon which the* Longitude *is counted from the* first Meridian.—*The Circles N E S, N R S, N C S, and N D S, are* Circles of Longitude *passing over severall Places.*——*The lesser* Circles *E, C, D, and V, are Circles or* Parallels of Latitude.

—*The Points* *, *, *, &c. *are severall Places whose distance we are to find by the following* Propositions.—*And the Points* ⊙,⊙,⊙, &c. *are the Poles of the Arches of great Circles which pass through the respective Places whose distance is to be found.*

And here note, *That the* Circles of Longitude *in this* Projection *are the same as the* Azimuth Circles *in the former* Projection; *and the* Centres *and* Poles *of them are found in the same manner.*———*Likewise, the* Parallels of Latitude *in this* Scheme *are the same with the* Parallels *or* Circles of Altitude *in the former* Projection, *and their* Centres *are found in the same manner as is before, in the Description of that* Projection, *prescribed; and therefore it shall here need no more Precepts for its Delineation, but we will proceed to the* Propositions *which shew how to find the* Distance of Places.

PROP. I.

Two Places which differ onely in Latitude, *to find their* Distance.

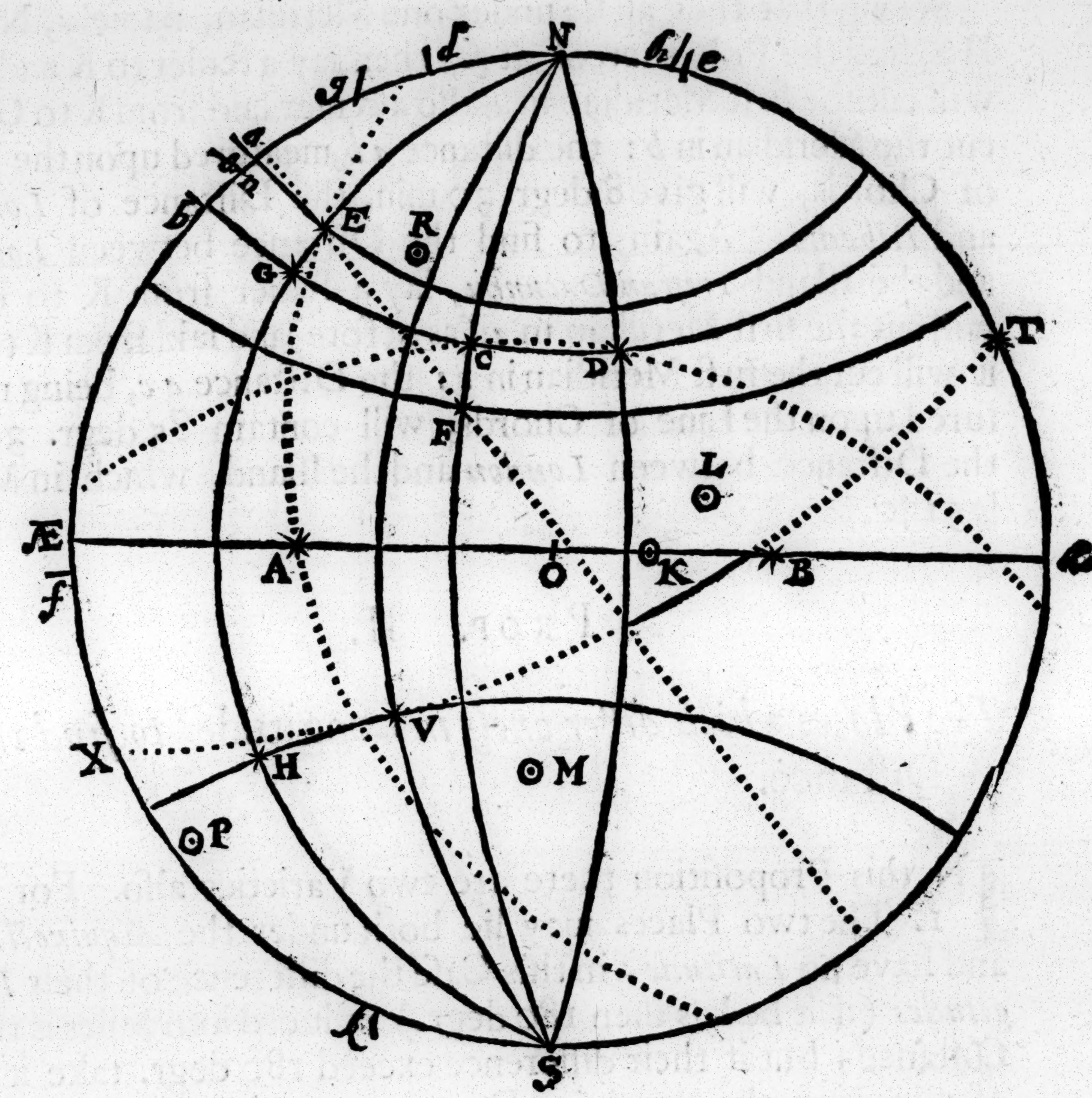
IN this Proposition there are two Varieties.

1. If both the *Places* lie under one and the same *Meridian*, and on one and the same Side of the *Æquinoctial*, either on the *North* or *South* Side thereof, then *substract* the *lesser Latitude* from the *greater*, and the Difference converted into *Miles* (by allowing 60 *Miles* to *one Degree*) shall give you the Distance.

Example. *London* and *Ribadio* lie both under one *Meridian*, namely of 20 degr. of Longitude; but they differ in Latitude, for *London* hath 51 d. 30 min. and *Ribadio* hath Latitude 43 d. both *North*; the difference of Latitude is 8 degr. 30 m. which being turned into Miles makes 510 miles.

2. If the two Places lie under one and the same *Meridian*, but

but one on the *North*, and the other on the *South*-side of the *Æquinoctial*, adde both the Latitudes together, the Sum is the Distance.



Example. *London* and the Island *Tristan Dacunhu* lie both under one *Meridian*; but *London* hath 51 degr. 30 min. *North* Latitude, and the Island hath 34 d. *South* Latitude: their Sum is 85 degr. 30 min. which converted into Miles (by dividing

the Degrees by 60. and allowing for every Minute one Mile) makes 5130 miles. And such is the distance of *London* and the Island *Tristan Dacunhu*.

To find the distance of these Places upon the Projection.

Seeing that they all lie under one Meridian, namely, N E G H S, find the Pole thereof at K; then lay a Ruler to K and E, it will cut the first Meridian in *a*; also a Ruler laid from K to G will cut the Meridian in *b*: the distance *a b*, measured upon the Line of Chords, will give 8 degr. 30 min. the Distance of *London* and *Ribadio*. Again, to find the Distance between *London* and the Island *Tristan Dacunhu*, lay a Ruler from K to E, it will cut the first Meridian in *a*, (as before) and laid from K to H, it will cut the first Meridian in *c*: the Distance *a c*, being measured upon the Line of Chords, will contain 85 degr. 30 m. the Distance between *London* and the Island, which in Miles is 5130.

PROP. II.

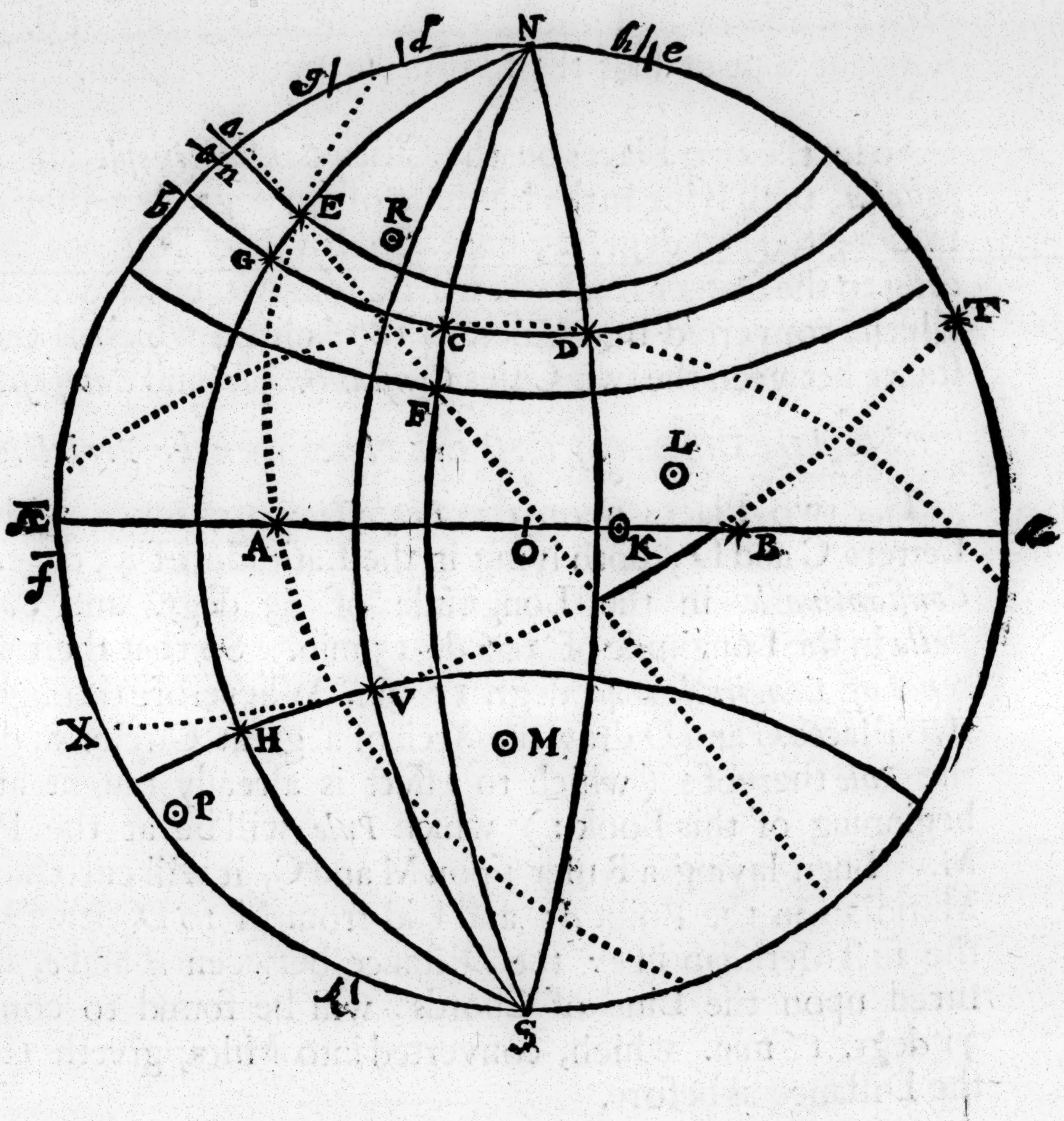
Two Places which differ onely in Longitude, *to find their* Distance.

IN this Proposition there are two Varieties also. For 1. The two Places may lie both under the *Æquinoctial*, and have no *Latitude*: in this Case the difference of their *Longitudes* (if it be less then 180 degr.) reduced into Miles is their Distance; but if their difference exceed 180 degr. take it out of 360 degr. the remaining Degrees turned into Miles will be the Distance of the two Places.

Example. The Island *Sumatra* and the Island of *S. Thoma* lie both under the *Æquinoctial*, the Island of *S. Thoma* having 33 d. 10 m. of Longitude, and the Island *Sumatra* 137 d. 10 m. The lesser Longitude taken from the greater leaves 104 d. 0 m.

which

which converted into Miles is 6240. And that is the Diſtance of the two Iſlands.



2. But if the two Places differ onely in *Longitude*, and lie not under the *Æquinoctial*, but under ſome other intermediate *Parallel of Latitude*, between the *Æquinoctial* and one of the *Poles*, then to find their Diſtance, this is

The

The Analogie or Proportion.

As the Radius 90 degr. is to the Co-sine of the common Latitude 47 degr.
So is the Sine of half the difference of Longitude 21 d. 37 m. to the Sine of half their Distance 15 degr. 38 m.

So let the two Places be the Cities *Constantinople* and *Compostella*, both lying in the Latitude of 43 degr. but they differ in Longitude 43 degr. 15 min. So that their Distance according to the former Analogie will be found to be 31 degr. 16 m. which, converted into Miles, is 1876 miles, which is the Distance between the two Cities *Constantinople* and *Compostella.*

To find the Distance of these two Places upon the Projection.

The two Places upon the *Projection* are noted with the Letters C and D, both lying in the Latitude of 43 degr. but *Constantinople* in the Longitude of 63 degr. and *Compostella* in the Longitude of 106 d. 15 min. So that their *difference of Longitude* is 43 degr. 15 min. Wherefore through the two Places C and D draw the Arch of a great Circle, and find the *Pole* thereof; (which to effect is already taught at the beginning of this Book:) which *Pole* will be at the Point M. Then laying a Ruler upon M and C, it will cut the first Meridian in the Point *d*; and laid from M to D, it will cut the first Meridian in *e*: the Distance between *d* and *e*, measured upon the Line of Chords, will be found to contain 31 degr. 16 min. which, converted into Miles, giveth 1876, the Distance as before.

PROP.

Prop. III.

Two Places differing both in Longitude *and* Latitude *being proposed, to find their* Distance.

THERE are three Varieties contained in this *Propositi-on.* For

1. One of the Places may lie under the *Æquinoctial*, and have no *Latitude*; and the other under some *Parallel of La-titude*, between the *Æquinoctial* and one of the *Poles*. For finding the Distance of Places that are so situate, this is

The Analogie or Proportion.

As the Radius 90 degr. is to the Co-sine of the difference of Longitude 76 degr. 50 min.

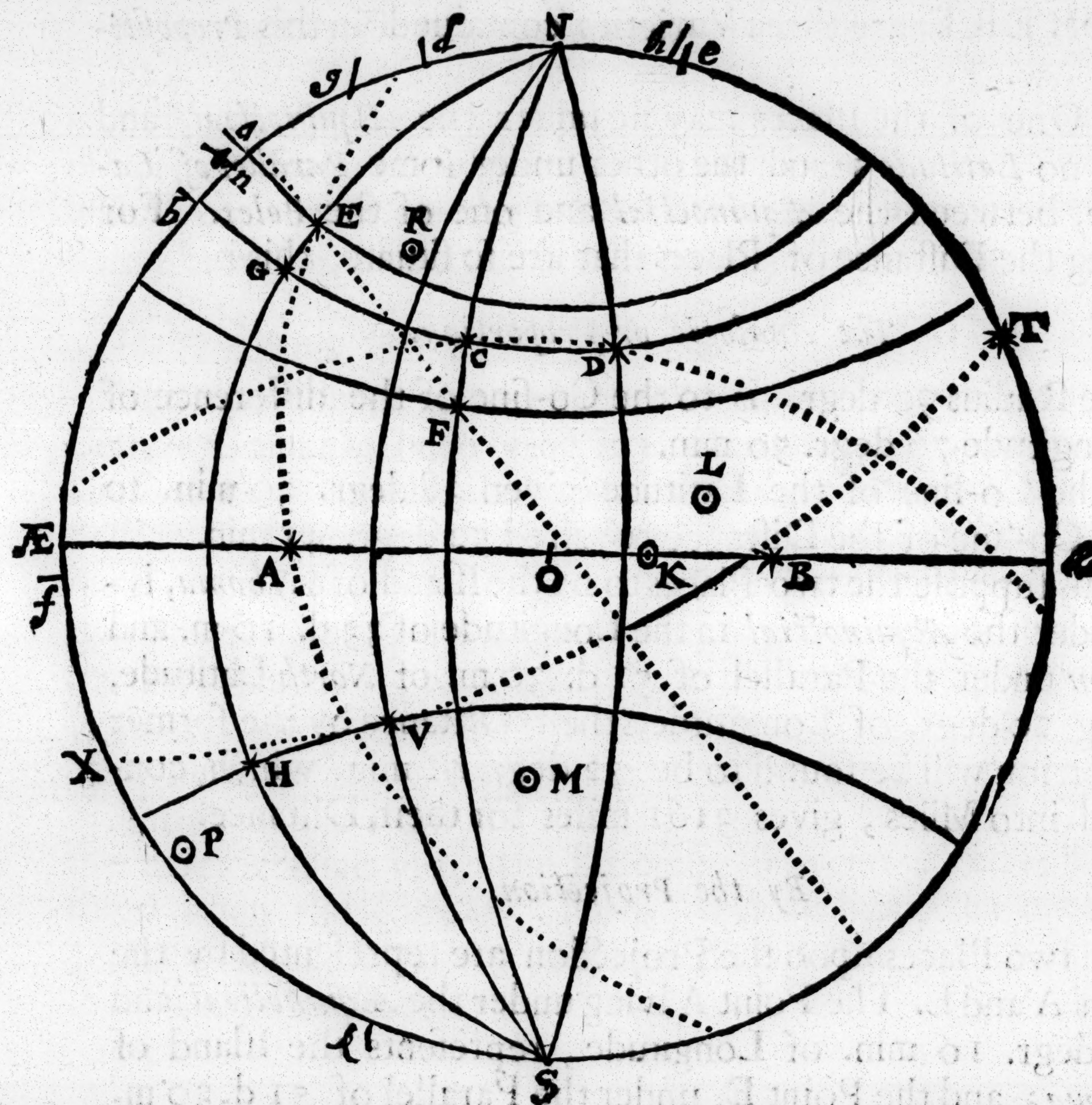
So is the Co-sine of the Latitude given 38 degr. 30 min. to the Co-sine of the Distance required 52 degr. 41 min.

Thus suppose the two Places to be the Island of *S. Thoma*, lying under the *Æquinoctial* in the Longitude of 33 d. 10 m. and *London* under the Parallel of 51 d. 30 m. of *North* Latitude, having 20 degr. of Longitude, their Distance by the former Proportion will be found to be 52 degr. 41 min. which, converted into Miles, gives 3161 miles for their Distance.

By the Projection.

The two Places upon the Projection are represented by the Letters A and E. The Point A lying under the *Æquinoctial*, and in 33 degr. 10 min. of Longitude, represents the Island of *S. Thoma*; and the Point E, under the Parallel of 51 d. 30 m. *North*, and in Longitude 20 degr. represents *London*.—— Through the two Places A and E (according to former Directions) draw an Arch of a great Circle, and find the *Pole* thereof, which will be at the Point L. A Ruler laid to L and

the Point E will cut the first Meridian in *n*; and the Ruler being laid from L to A will cut the first Meridian in *f*: the Distance *n f*, being measured upon the Line of Chords, will be found to contain 52 degr. 41 min. as before, which in Miles is 3161.



2. If both the Places proposed shall be without the *Æquinoctial*, but on one Side, either both towards the *North*, or both

both towards the *South*, the finding of their Distance is by this

Analogie or Proportion.

(1.) As the Radius 90 degr. is to the Co-sine of the difference of Longitude 44 degr.

So is the Tangent of 38 degr. 30 min. to a fourth Tangent 28 degr. 55 min. which taken from the Complement of the Lat. of *Jerusalem* 58 degr. 20 min. leaves 29 d. 25 m.

(2.) As the Co-sine of the fourth Tangent 61 degr. 35 min. is to the Co-sine of 60 degr. 35 min.

So is the Co-sine of the Latitude of *London* 38 degr. 30 min. to the Co-sine of the Distance 51 degr. 9 min.

So the two Places propounded being *London*, lying in *North* Latitude 51 degr. 30 min. and Longitude 20 degr. and the other, *Jerusalem*, lying in *North* Latitude also 31 degr. 40 min. and Longitude 66 degr. you may find their Distance by the foregoing Analogie to be 38 degr. 51 min. which in Miles makes 2331.

By the Projection.

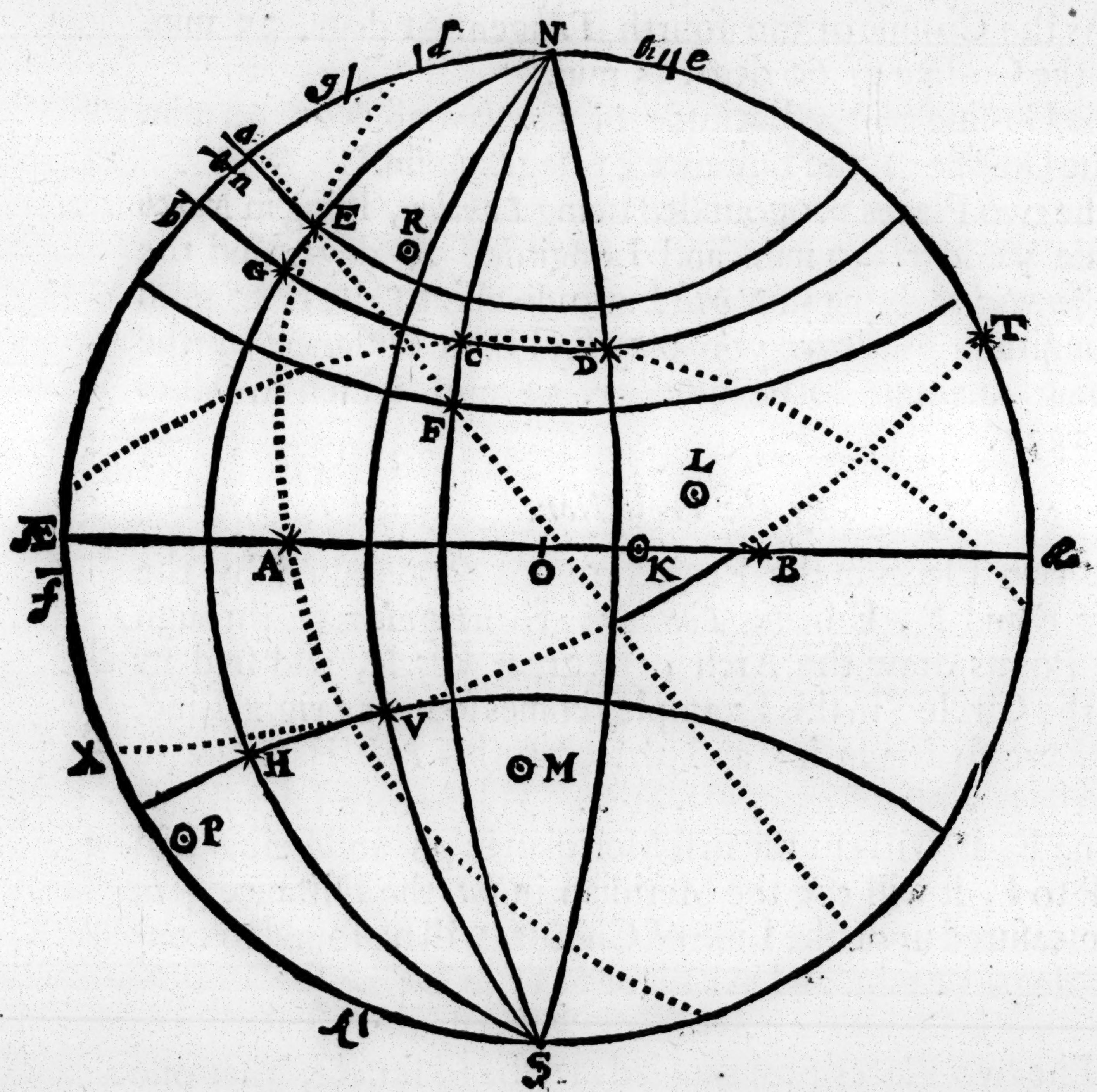
The two Places in the Projection are represented by the Letters E and F, E being *London*, F *Jerusalem*; through which Points draw the Arch of a great Circle, and find its *Pole*: the Circle (in this Example) comes so near a right Line, that I have so drawn it; and therefore his *Pole* is but little within the outward Circle, viz. at P. Wherefore lay a Ruler to P and E, it will cut the first Meridian in *g*; and being laid from P to F, it will cut the Meridian in *h*: the Distance *g h*, being measured upon the Line of Chords, will be found to contain 38 degr. 51 min. and in Miles 2331, as before.

3. The two Places propounded may be so situate, that one of them may lie on the *North*, and the other on the *South*-side of the *Æquinoctial*. For finding the Distance of such Places follow this

Analogie or Proportion.

(1.) As the Radius 90 degr. is to the Co-sine of the difference of Longitude 40 degr.

So is the Co-tangent of the greater Latitude 50 d. to the Tangent of a fourth Arch 37 d. 10 m. which being substracted out of the other Latitude, and 90 d. added thereto, say,



(2.) As

(2.) As the Co-sine of the Arch found 52 degr. 50 min. is to the Co-sine of the Arch remaining 52 degr. 50 min.
So is the Co-sine of the Latitude first taken 50 degr. 00 min. to the Co-sine of the Distance 40 degr. which taken from 180 degr. there remains 140 degr. for the Distance of the two Places.

So the two Places propounded being the *Cape of Good hope*, lying in the Latitude of 40 degr. *South*, and Longitude 50 d. and the other Place *Malibrigo*, lying in 26 degr. of *North* Latitude, and in 180 degr. of Longitude, you may find their Distance by the foregoing Analogie to be 140 degr. which, converted into Miles, make 8400. And such is the Distance of the two Places.

By the Projection.

In the Projection the two Places are represented by the Letters T and V; the Letter V representing the *Cape of Good hope*, and T *Malibrigo*. Now *Malibrigo* having 180 degr. of Longitude, (which is just half the Circumference of the *Æquinoctial*, and is as far remote as any Place can be from the first Meridian; for if you were to project any Place having above 180 d. Longitude, (as suppose 230 degr.) you must substract such Longitude from 360 degr. and project the remainer; so 230 degr. being taken from 360 degr. leaves 130 degr. which must be projected in stead of 230 degr. and by this means it is that *Malibrigo* is projected upon the outermost Circle or first Meridian)

Through these two Points T and V draw the Arch of a great Circle, T V X, and find its *Pole* at R: then a Ruler laid at R and the Point V will cut the first Meridian in *k*, and T *k*, being measured upon your Line of Chords, will be found to contain 140 d. and that is their Distance, which in Miles maketh 8400.

These are all the Varieties *of* Positions *of* Places *upon the* Terrestriall Globe; *for no two Places (whose Distance can be*

required) can be proposed, but they must fall under one or other of the Varieties *contained in some of these three* Propositions. *And note that this way of finding* the Distance of Places *is the most* absolute *and* exact *of any other.*——*And what is here said concerning finding these* Distances *the ingenious may apply to* Circular Sailing, *of all other waies the most perfect: which I shall leave to the* industrious Sea-man *to find out of himself, till I present him with something of that kind: in the mean time let him make use of the foregoing* EXERCISES, *and this which follows.*

THE

The Doctrine of *RIGHT-LINED TRIANGLES* applied to Practice in NAVIGATION:

Whereby

Sundry *Nauticall Questions* are resolved; and many *Problems* of *Sailing*, both by the *Plain* and *Mercator*'s Chart, performed by *Protraction*, by *Calculation*, and also wrought upon the *Chart* it self.

The Ninth EXERCISE.

SECTION I.

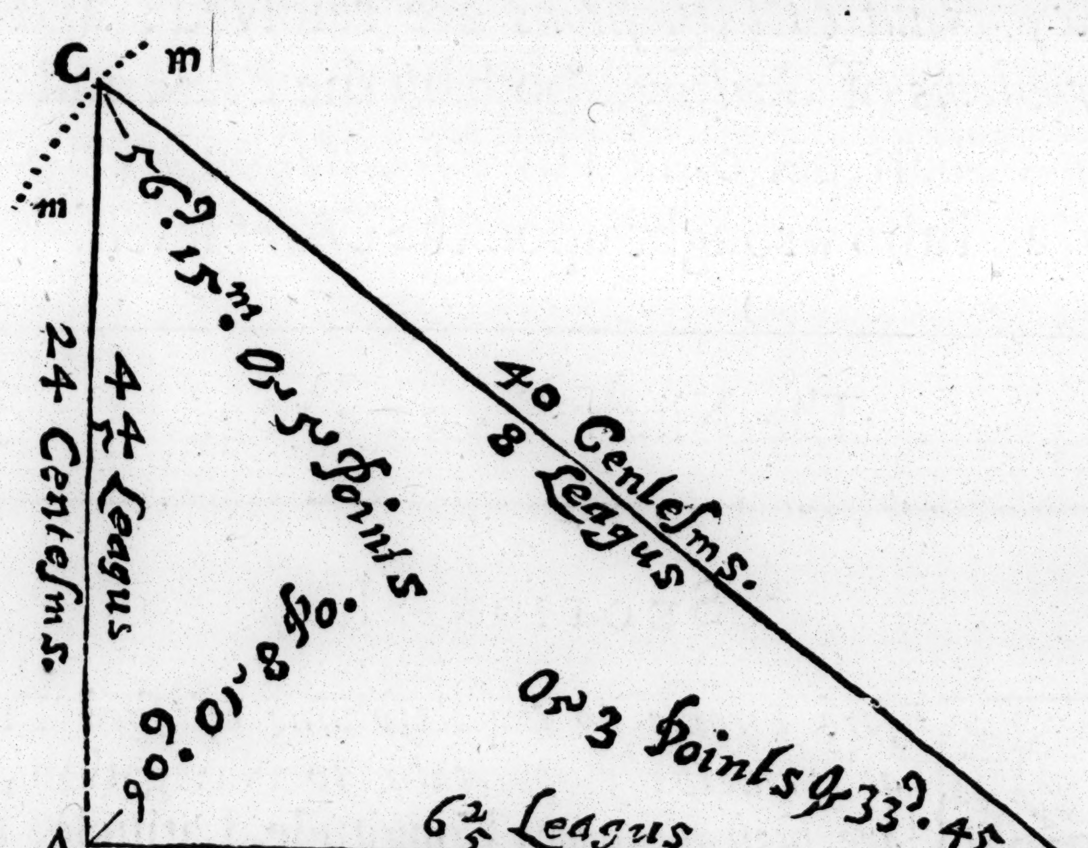
BEfore I come to the Resolving of such *Problems as principally appertain to* Navigation, *which are such as concern* Longitude, Latitude, Rhumb *and* Distance; *I shall shew how the Solution of* plain Triangles *may be made applicable to the taking of* Heights *and* Distances, *and so (in the first place) propose and work severall* Nauticall Questions, *which to the industrious*

strious Mariner *will be both* delightfull *and* profitable, *and give occasion to him to invent and put in Practice others of his own contrivance.*

Question I.

There are two Ships set sail from the Port A, the one saileth directly North 24 *Centesms, (or* 4 *Leagues and* $\frac{4}{5}$ *parts of a League,) and the other directly East* 32 *Centesms, (or* 6 $\frac{2}{5}$ *Leagues;) I demand how the two Ships bear one from the other, and also how far they are asunder.*

DRAW a right Line A B, and upon A raise the Perpendicular A C: let the Point A represent the Port from whence the two Ships set sail: then, because the first Ship



sailed 24 Centesms North, from a Scale of equal parts take 24 Cent. and set them from A to C; so shall C be the place of the first Ship. Then, because the other Ship sailed directly East,

East, which is a Quadrant or Quarter of the Compass distant from the North, therefore the Angle at A must be a right Angle: And because the second Ship sailed East 32 Cent. take 32 Cent. from the same Line of equal parts, and set them upon the Line A B, from A unto B; so shall B be the place of the second Ship.

Now first, To know how these two Ships bear one from another, Draw the Line C B, and measure the Quantity of the Angle at B, which you shall find to be 33 degr. 45 min. which is three Points from the West Northerly, that is the N. W. by West Point of the Compass; and so doth the second Ship B bear from the first Ship C.——Again, find the quantity of the Angle at C, which you shall find to be 56 degr. 15 m. which is five Points from the South Easterly, that is the S. E. by East Point of the Compass; and so do the two Ships bear one from the other.——Then for the Distance that the two Ships are from each other, Take in your Compasses the distance between B and C, which measure upon your Scale of equal parts, and you shall find it to contain 40 Centesms or 8 Leagues; and so far asunder are the two Ships B and C.

The Bearing of the Ships one from the other is found by the first Case of Right-angled plain Triangles by this Analogie.

As the Distance that the first Ship sailed is to the Distance that the second Ship sailed;
So is the Radius to the Tangent of the Angle that the first Ship bears to the second. The Complement whereof is the bearing of the second Ship to the first.

The Distance of the Ships from each other is found by the seventh Case, by the following Analogie. Having found the Angle C, the bearing of the first Ship from the second, say,

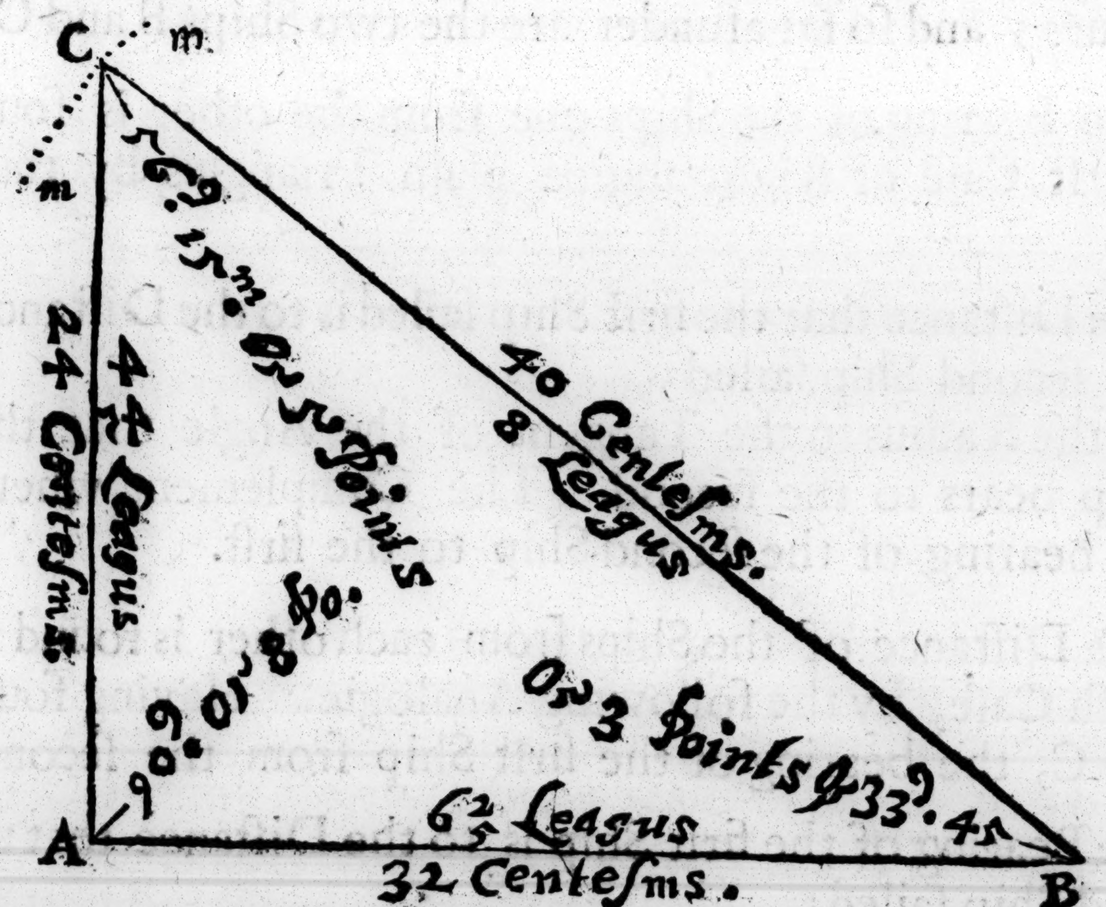
As the Bearing of the first Ship is to the Distance that the second Ship sailed;

So is the Radius to the Distance of the two Ships.

Quest. II.

A Ship at A discovers an Island at C, lying from her directly East, but she sails from A towards B 32 Cent. or $6\frac{2}{5}$ Leagues directly South; but her Compass coming to some mischance, that use cannot be made of it, she again at B discovers the same Island, and sails upon an unknown Point of the Compass directly upon the Island, and touches upon it, having sailed 8 Leagues.——I demand upon what Point of the Compass the Ship sailed from B to C, and also how far off the Island was from A, where it was first discovered.

DRAW a Line C A, representing a Line of East and West, and upon A erect a Perpendicular A B, and from A to B set off 32 Cent. or $6\frac{2}{5}$ Leagues, the distance that the Ship sailed from A to B. Then take out of your Scale of equal



part

parts 40 Cent. or 8 Leagues, the diſtance that the Ship ſailed from B to the Iſland; and ſetting one foot of the Compaſſes in B, with the other deſcribe an obſcure Arch of a Circle *m m*, croſſing the Eaſt and Weſt Line in C: ſo is C the place of the Iſland.

Now firſt, to find upon what Point of the Compaſs the Ship ſailed from B to the Iſland, you muſt find the quantity of the Angle at B, (either by your Line of Chords, or Protracting Quadrant,) and you ſhall find it to contain 33 degr. 45 min. which is three Points from the North Eaſterly, that is N. E. by N. and upon that Point did the Ship ſail from B to the Iſland at C.——Then, to know how far the Iſland C was from A, where it was firſt diſcovered, Take in your Compaſſes the length of the Line A C, and meaſure it upon your Scale; ſo ſhall you find that to contain 24 Cent. or $\frac{4}{5}$ Leagues: and ſo far diſtant was the Iſland from A.

The Point of the Compaſs that the Ship ſailed upon from B to C may be found by the ſecond Caſe of Right-angled plain Triangles, by this Analogie.

As the Diſtance which the Ship ſailed from B to C is to the Radius;

So is the Diſtance ſailed between A and B to the Co-ſine of the Point that the Ship ſailed upon from B to C.

The Diſtance that the Ship was from the Iſland, when firſt diſcovered, may be found by the fifth Caſe of Right-angled plain Triangles, by the following Analogie.

(1.) As the Diſtance that the Ship ſailed from B to C is to the Radius;

So is the Diſtance that the Ship ſailed from A to B to the bearing of the Iſland from B.

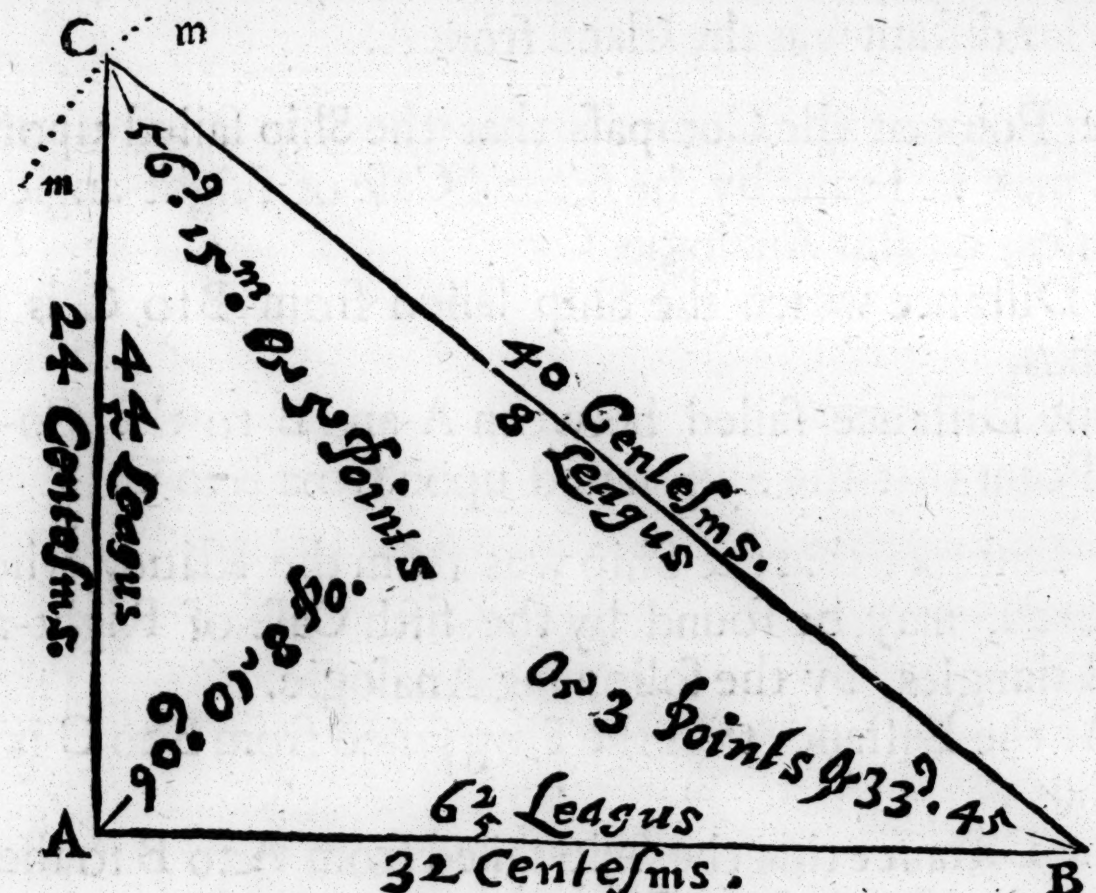
(2.) As the Radius is to the Diſtance that the Ship ſailed from C to B;

So is the Sine of the Rhumb that the Ship ſailed upon from B to C to the Diſtance of the Iſland from A.

Quest. III.

There are two Ports at A and B which are distant 6 $\frac{2}{5}$ Leagues, and lie directly North and South of each other; from whence two Ships set sail, both for the Port C: the Ship at B sails away upon a South-W. by South Point, and the Ship at A sails directly West.——I demand how many Leagues either of the Ships had sailed when they met at the Port C, and also how the Port C did bear from that at B.

DRAW a right Line A B, and upon it set off 32 Centesms, or 6 $\frac{2}{5}$ Leagues. Now because the Ship at B steered a S. W. by S. Course, which is three Points from the



South-Westerly, therefore upon the Point B protract an Angle of 33 degr. 45 min. and draw the Line B C.——Then, because the Ship at A steered a Westerly Course, which is a Quarter from the North, upon the Point A protract an Angle of

of 90 degr. and draw the Line A C, cutting the former Line B C in C.——— Now to know how many Leagues each Ship sailed, take in your Compasses the length of the Line B C, and measuring it upon your Scale, you shall find it to contain eight Leagues; and so many did the Ship that came from B sail. Also take the length of the Line A C in your Compasses, and measuring that upon your Scale, it will be found to contain 24 Centesm. or $4\frac{4}{5}$ Leagues; and so much did the Ship that came from A sail. Now to know how the Port at C did bear from that at B, find the quantity of the Angle at C, which you shall find to be 56 degr. 15 min. that is, five Points from the East Northerly, namely, N. E. by N. and so did the Port C bear from B.

The finding of the Distance that each Ship sailed may be done by the third Case of Right-angled plain Triangles by this Analogie.

As the Distance of the two Ports A and B is to the bearing of the Port C from B;
So is the Sine of the Rhumb that the Ship sailed upon from B to C to the Distance that the Ship sailed from A to C;
And so is the Radius to the number of Leagues that the Ship sailed from B to C.

Quest. IV.

A Ship at C discovers a Point of Land at A bearing from her S. S. E. but she shapes a Course E. by S. and sails away 8 Leagues to B, and at B she discovers the same Point of Land bearing from her W. S. W.——— I demand how far the Ship was from Land being at C and B.

DRAW a Line C B containing 40 Cent. or 8 Leagues, and upon C protract an Angle of 56 degr. 15 min. or five Points, which is the difference the Point of Land did bear

from the Ship being at C, and the Point upon which he sailed from C to B; and draw a right Line C A.——Then upon the Point B protract an Angle of 33 degr. 45 min. which is the difference of the Ship's bearing from C and A, she being at B, namely, W. S. W. and draw the Line B A, cutting the Line C A, before drawn, in A.

Now to find how far the Ship was from Land being at C, measure the Line C A upon your Scale of equal parts, and you shall find it to contain 24 Centesms, or 4 $\frac{4}{5}$ Leagues: and so far was the Ship from the Land, when she was at C. Also measure the length of the Line B A, and you shall find that to contain 32 Cent. or 6 $\frac{2}{5}$ Leagues: and so far from Land was the Ship being at B.

To find these Distances by the Canon of Sines and Table of Logar. you may doe it by the fourth Case of Right-angled Triangles, by this Analogie.

As the Radius is to the Distance that the Ship sailed from C to B;

So is the Bearing of the Ship, being at C, to her Distanee from Land, being at B.

Or,

The Bearing of the Sip, she being at B, to her Distance from Land at C.

Quest. V.

A Ship being at A discovers two other Ships at C and B; the Ship at C bears from her directly East, and the other Ship at B bears from her directly South. The Ship at A sails directly South 32 Cent. to B, and being at B, steers away upon an unknown Course to C 40 Cent. or 8 Leagues.——I demand upon what Point the Ship sailed from B to C,——and also how far C is distant from A.

Draw

DRAW a right Line A B, for the bearing of the Ship B from the Ship A, which was direct South. Also from A draw another Line A C, for the bearing of the Ship C from the Ship A, which was directly East. Now because between the South and the East is 90 degr. or one Quarter of the Compass, therefore upon the Point A protract an Angle of 90 degr. drawing the Lines A C and A B at right Angles. This done, take 32 Cent. out of your Scale of equal parts, which is the distance that the Ship sailed South from A to B. Then take from the same Scale 40 Cent. which is the distance that the Ship sailed from B to C upon an unknown Point. And with this distance, setting one foot of the Compasses in B, with the other describe an obscure Arch of a Circle *m m*, cutting the Line A C in the Point C, and draw the Line C B.—Now to find upon what Point of the Compass the Ship sailed from B to C, find the quantity of the Angle at B, which you shall find to contain 33 degr. 45 min. that is three Points from the North Easterly, namely, N. E. by N. and upon that Point did the Ship sail from B to C.—Then to find how far C is distant from A, Take the Line C A in your Compasses, and measuring it upon your Scale, you shall find it to contain 24 Cent. or $4\frac{4}{5}$ Leagues: and so far is C distant from A.

The Point upon which the Ship sailed from B to C may be found by the second Case of Right-angled Triangles, by this Analogie.

As the Distance that the Ship sailed from B to C is to the Radius;
So is the Distance that the Ship sailed from A to B to the Cosine of the Rhumb from the Meridian.

Then for the Distance of C from A.

As the Radius is to the Distance that the Ship sailed from B to C;

So

So is the Rhumb from the Meridian that the Ship sailed upon from B to C to the Distance of C A.

Quest. VI.

Two Islands at A and C are discovered by a Ship at B, the Island A bears from the Ship at B N. N. W. and the Island at C bears N. by E. from B; the Ship being at B sails away N. N. W. to the Island A, and having sailed 32 Cent. touches upon the Island, and being there findes that the Island C bears from the Island A E. N. E.—I demand how far the Ship at B was from the Island C, and also how far the two Islands were asunder.

DRAW a Line A B, and upon it set 32 Cent. which is the Distance that the Ship sailed from B to the Island at A. And because the Island A did bear from B N. N. W. and the Island at C N. by E. which are three Points, or 33 degr. 45 min. asunder, upon the Point B protract an Angle of 33 d. 45 min. and draw the Line B C.—Then because the Island at C bears from the Island at A E. N. E. which is eight Points, or 90 degr. from N. N. W. upon the Point A protract an Angle of 90 degr. and draw the Line A C, cutting the Line B C in C.

Now to find the Distance of the Ship being at B from the Island C, take the Line C B in your Compasses, and applying it to your Scale, you shall find it to contain 40 Cent. or 8 Leagues; and so far was the Ship at B from the Island at C. And to find the Distance of the Islands one from the other, take C A in your Compasses, and measure it upon your Scale, you shall find it to contain 24 Cent. or $4\frac{4}{5}$ Leagues; and so far distant were the Islands one from the other.

The Distance from A to C may be found by the sixth Case of Right-angled plain Triangles, by this Analogie.

As

As the Co-ſine of the Rhumb that the Ship ſailed upon from B to A is to the Diſtance that the Ship ſailed from B to A;
So is the Radius to the Diſtance of the Ship at B from the Iſland at C.

Then for the Diſtance of the two Iſlands, by the fourth Caſe ſay,

As the Radius is to the Diſtance C B;
So is the Sine of the Difference between the bearing of the two Iſlands from B to the Diſtance of the two Iſlands C and A.

Quest. VII.

Two Ships ſet out from one and the ſame Port A; the Ship C ſails 24 Cent. or 4 $\frac{4}{5}$ Leagues directly Eaſt, and the Ship B ſails away 32 Cent. or 6 $\frac{2}{5}$ Leagues directly South.——When they have thus ſailed, I demand how far the two Ships are from each other.

DRAW a right Line A B, and ſet off upon it 32 Cent. the Diſtance that the Ship ſailed from A to B South. —Then becauſe the other Ship ſailed directly Eaſt, which is 90 degr. from the South, upon the Point A erect the Perpendicular A C, and upon it ſet off 24 Cent. or 4 $\frac{4}{5}$ Leagues from A to C, which was the Diſtance the other Ship ſailed Eaſt. ——Then draw the Line C B, which being taken in your Compaſſes, and meaſured upon your Scale, will be found to contain 40 Cent. or 8 Leagues. And ſo far are the two Ships from each other.

This Diſtance, by the ſeventh Caſe of Right-angled plain Triangles, may be found by this Analogie.

(1.) As the Diſtance that the Ship ſailed from A to B is to the Diſtance that the Ship ſailed from A to C;

X So

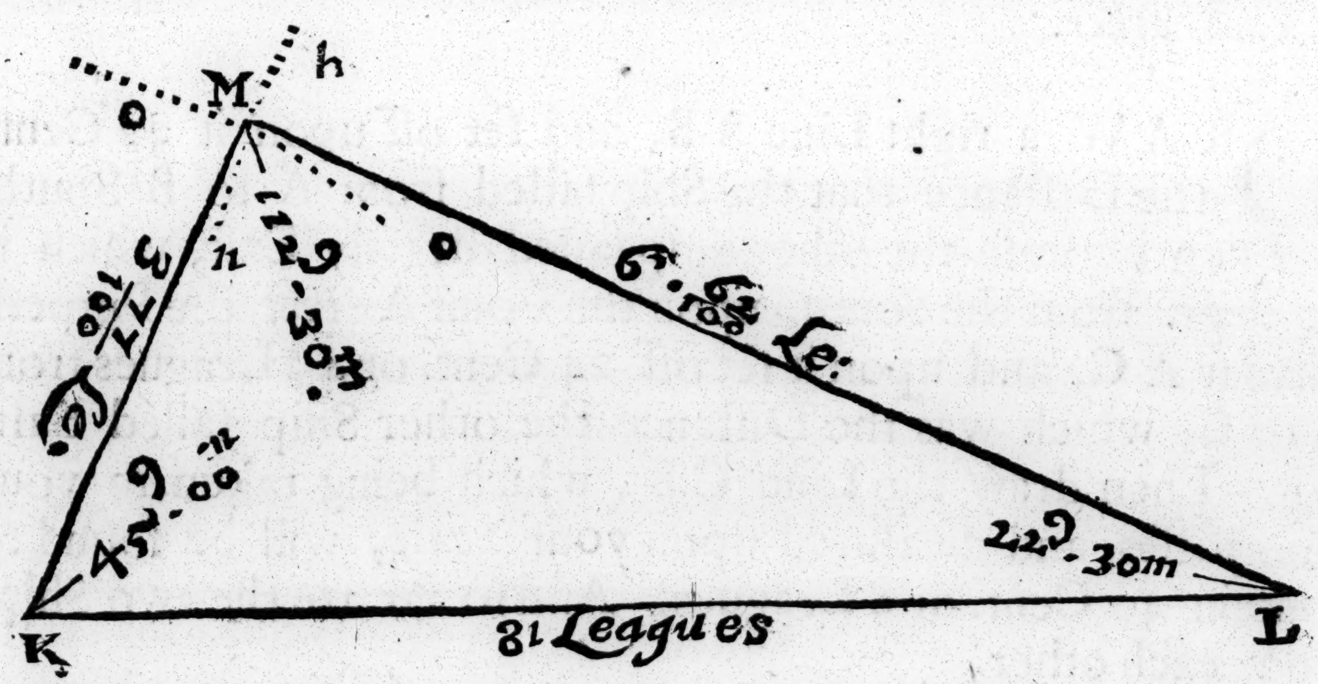
So is the Radius to the Tangent of the Angle at B.

(2.) As the Sine of the Angle at B is to the Distance C A; So is the Radius to the Distance C B.

Quest. VIII.

Two Ships set sail from the Port at K; the one sails $3\frac{77}{100}$ Leagues upon the S. W. Point towards M, the other sails 8 Leagues upon the West Point towards L.—I demand how many Leagues the Ships at M and L are asunder, and also how the Ship at M bears from the Port K, and the other Ship at L.

DRAW a right Line K L, and by help of your Scale set off upon it 8 Leagues, the Distance that the Ship sailed from K to L upon the West-Point. Then because the other Ship sailed $3\frac{77}{100}$ Leagues from K towards M, upon the



S. W. Point, which is 45 degr. or 4 Points from the West, therefore upon the Point K protract an Angle of 45 degr. and draw the Line K M, setting off upon it from K to M $3\frac{77}{100}$ Leagues, the Distance that the Ship sailed from K to M, and draw the Line M L.

Now

Now to know, First, how far distant the Ships at M and L are from each other, take in your Compasses the length of the Line M L, which applie to your Scale, and you shall find it to contain 6 $\frac{62}{100}$ Leagues.—And, Secondly, to find how the Ship at M bears from the Port K and the other Ship at L, you must find the quantity of the Angle at M, which you will find to be 112 degr. 30 min. that is, eleven Points. Now because the Course from K to M was S. W. therefore the Ship at M bears from the Port K N. E. And seeing that the Angle at M is 112 degr. 30 min. or eleven Points; therefore eleven Points counted from the N. E. Point is the Bearing of the Ship at M from that at L, which is W. N. W.

The Distance of the Ships M and L may be found by the fifth Case of Oblique-angled plain Triangles; and the Bearings by the second Case.

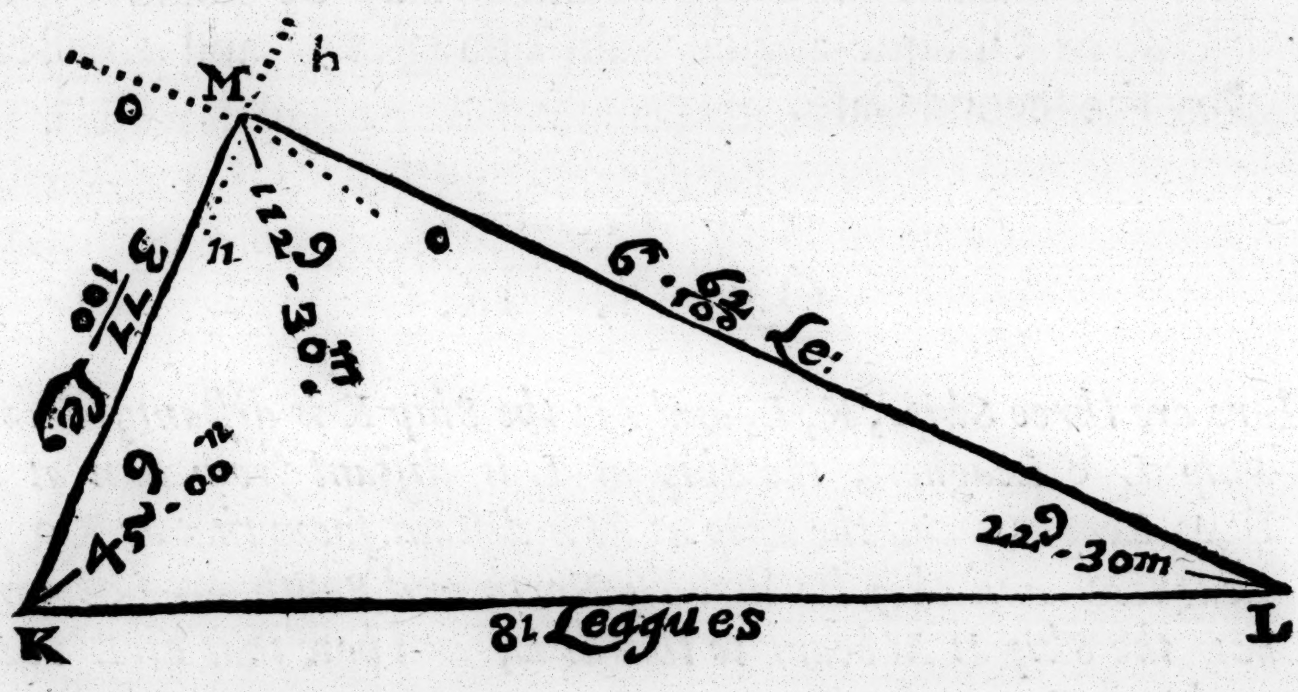
QUEST. IX.

There are three Ships, K, L, and M: the Ship K is distant from the Ship L 8 Leagues; the Ship at L is distant from that at M 6 $\frac{62}{100}$ Leagues; and the Ship at M is distant from that at K 3 $\frac{77}{100}$ Leagues; and they lie directly North and South.—I demand how the Ship at M bears to that at L, and how that at L bears to that at K.

DRAW a right Line, and out of your Scale take 8 Leagues, and set them thereon from K to L, for the Distance of the Ships at K and L. Then take 3 $\frac{77}{100}$ Leagues, the Distance of the Ships K and M, out of your Scale; and setting one foot of the Compasses in K, with the other describe the obscure Arch of a Circle *o o*. Again, take 6 $\frac{62}{100}$ Leagues from your Scale, which is the Distance that the Ship L was from the Ship M; and setting one foot of the Compasses in L, with the other describe the obscure Arch of a Circle *n n*, crossing

the former Arch in the Point M. Then draw the Lines M K and M L; so have you their true Positions.

Now to find their Bearing one from another; forasmuch as the Ships M and K did lie North and South of each other, find the quantity of the Angle at M, which is 112 degr. 30 min. that is, eleven Points from the South Eastward, (or 3 Points from the East Northward,) either of which will be the N.E. by E. Point: and so doth the Ship M bear from that at K. And for the Bearing of that at K from that at L, finde the quantity of the Angle at L, which will be 22 degr. 30 min. or two Points; so two Points from the S. W. by W. Point Southward is S. W. by S. and so doth the Ship L bear to that at K.



The Bearings of the Ships from each other may be found by the third Case of Oblique-angled plain Triangles, by the Analogie in that Case set down.

PRO-

PROBLEMS Of Sailing by the Plain Sea-Chart.

SECTION II.

AMONG Sea-men *there are* Three *principal waies of* Sailing *most in* Use *and* Practice: *Two whereof are* Rectilineal, *performed by* Right Lines, *the Third is* Sphericall *or* Circular, *performed by* Arches *of* great Circles *of the* Sphere.

Of the Two first, the one is called Plain Sailing, *or,* Sailing *by the* Plain Sea-Chart.

The other is called Mercator*'s* Sailing, *or,* Sailing *by* Mercator*'s* Chart.

These two Charts *are both of them composed of* Right Lines, *yet differ both in their* Construction *and* Use, *though not so much in their* Use *as in their* making *or* composition.

The Plain Sea-Chart *consisteth of* Meridians *and* Parallels, *which are drawn in all parts equal from the* Æquinoctial *towards either of the* Poles; *which is erroneous, as hereafter shall be discovered.*

Mercator's Chart *hath the Degrees of* Longitude *in every* Parallel of Latitude *equal to those in the* Æquinoctial, *as the* Plain Chart *hath: But the Degrees of* Latitude *do increase more and more (as they grow nearer the* Poles) *in such a Proportion as every* Parallel of Longitude *doth decrease.*

The way of Sailing by the Plain Sea-Chart *is much in use, nay too much, considering the* Errours *that it leads* Sea-men *into; though they are not so easily discovered in* short *as in* long Voiages, *nor in* places near *the* Æquinoctial *as those* nearer *the* Poles. *But, I suppose, it is more used, for the ease there is in* Projecting *of this* Chart, *more then in that of* Mercator's*: otherwise I know not why that should be so as it is embraced, and the other (I mean that of* Mercator's*) so much neglected; which comes so near to the* Spherical *way of* Sailing, *that there is an insensible difference between them. But I shall bring them* face *to* face, *that the* ingenious Sea-man *may see their Difference, and thereby abandon* Errour, *and embrace the* Truth. *For in the following* Problems *I shall perform the* same *thing by* both Charts, *by which the* Errours *may more palpably be discovered. And to retain the* Method *which I have observed in all the foregoing* EXERCISES, *I shall shew how these* Nauticall Problems *may be* Trigonometrically *performed by the* Tables *of* Logarithms, *and* Canons *of* Artificial Sines *and* Tangents. *And to begin this* Exercise, *the first thing that I shall propose unto you is*

The Making of the Plain Sea-Chart.

A *Sea-Chart* may be made either *general*, or *particular*. A *General Sea-Chart* is that whose Degrees of *Latitude* proceed from the *Æquinoctial* to either *Pole*, which in the *common Sea-Chart* may be done; but it will be egregiously false, as the Degrees of *Latitude* grow nearer the *Pole*, as I have already declared.——A *Particular Sea-Chart* is such a one as is made properly for one particular *Navigation:* as if your whole

whole *Navigation* were not to exceed the *Latitudes* of 48 and 60 degr. of *Latitude*, and not to differ in *Longitude* above 8 degrees.

Now to *project* or *make* ſuch a *Chart*; Firſt, draw a right Line A B, repreſenting the *Meridian*, and croſs it at right Angles in the Point A with another right Line A D, repreſenting the *Parallel* of your leaſt *Latitude*, namely, of 48 degr.——— Secondly, conſider what Diſtance you will have your *Parallels of Longitude* and *Latitude* to be, (for in this *Chart* they are both equal,) whether an Inch, 2, 3, or 4 Inches, (for the larger the better.) But in this Example I have made them onely half an Inch. I take therefore half an Inch out of an exact *Scale*, and run it up upon the *Meridian Line* A B, from A to 49, from 49 to 50, from 50 to 51, &c. till I come to my greateſt *Latitude*, which is here ſuppoſed to be 60 degr.——Thirdly, run the ſame Diſtance of half an Inch from A towards D, upon the Line A D, eight times, becauſe the Difference of *Longitude* in your whole *Navigation* will not exceed 8 degrees.——— Fourthly, draw the Line C D, parallel to A B, and B C, parallel to A D, and run the ſame Diſtances upon the Line B C as are upon the Line A D, and the ſame upon C D as are upon the Line A B.———Fifthly, from each Degree of *Latitude* in the Line A B draw to the like Degree of *Latitude* in the Line C D a right Line, as 49, 49; 50, 50; 51, 51; 52, 52; &c. till you have drawn all your *Parallels* of *Latitude*.——Sixthly, for your *Meridians*, they are to be drawn in like manner as were the *Parallels* of *Latitude*, all of them equidiſtant, and parallel to your firſt *Meridian* A B, as the Lines 1, 1; 2, 2; 3, 3; &c. And by this means have you the *Meridians* and *Parallels* drawn.

The *grand Diviſions*, or *whole Degrees*, being thus ſet upon your *Chart*, we now come to ſub-divide them. And for the dividing of the Degrees of the *Æquinoctial* at the top and bottom of your *Chart*, let each of them be divided into 5 or 10 parts, and each of thoſe parts ſub-divided into 5 or 10

more

more leſs parts, according as Quantity will permit; for every one of them is ſuppoſed to be divided into 100 or 1000 parts.

For the dividing of the *Degrees of Latitude*; they may be divided as thoſe of *Longitude* were, into 100 parts. But ſometimes each Degree is ſubdivided into 60 *Minutes*, or *Engliſh Miles*, or into 20 *Leagues*.—Now I have divided the *Degrees of Latitude* in this *Chart* each of them into 5 parts, by which means it is capable of the *Numeration* either by *Miles*, *Leagues*, *Centeſms*, or 100 parts.———For if you count by 60 *minutes*, or *miles*, then every of thoſe Diviſions will be 12 *minutes*, or *miles*; if by 20 *Leagues*, then every Diviſion will contain 4 *Leagues*; and if by *Centeſms* or 100 parts, then every of them is 20 *Centeſms*. And thus much concerning the *Making* or *Projecting* of this *Chart*. I now come to ſhew

Some Uſes *of the* Plain Sea-Chart.

THE Problems that are to be reſolved by (or upon) the Sea-Chart are chiefly ſuch as concern *Longitude*, *Latitude*, *Rhumb* or *Courſe*, and *Diſtance*.

Longitude *is the Diſtance of a Place from ſome known* Meridian *to that Place, and is alwaies counted upon the* Æquinoctial.

Latitude *is the Diſtance of any Place from the* Æquinoctial, *counted upon that* Meridian Circle *which paſſeth over that Place.*

Rhumb *or* Courſe *is the* Angle *that a Ship in his Sailing makes with the* Meridian, *and is diſcovered in the general by the* Magneticall Needle, *which alwaies reſpecteth the* North; *and (though not directly, yet) its* Variation *being often obſerved, and the* Chart *rectified thereby, (as I have before ſhewed how it may be done by ſeverall means) is the beſt help that* Navigators *yet have to ſteer their* Courſe *by.*

Diſtance *is the number of* Leagues, Miles, *or* Centeſms, *that any Ship hath* ſailed.

Raiſing

Raising of the Pole *is when a Ship sails from a* lesser *to a* greater Latitude.

Depressing of the Pole *is sailing from a* greater *to a* lesser Latitude.

These Terms thus explained, I will proceed to Practice, as followeth.

PROBL. I.

How to set any Place upon your Chart *according to its Longitude and Latitude.*

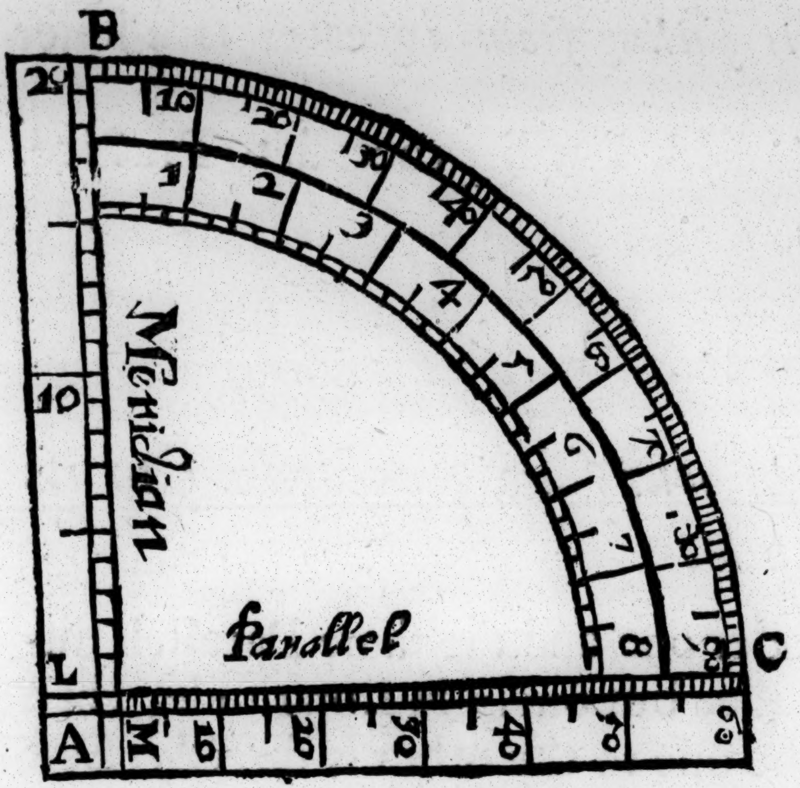
IF the two Places lie under one and the same Parallel, differing not at all in Latitude, but onely in Longitude, then the Course leading from the one to the other is directly East or West. As E and F are two Places lying under the Parallel of 50 degr. of Latitude, and differ in Longitude $5\frac{1}{2}$ deg. Lay a Ruler to $5\frac{1}{2}$ degr. both at the top and bottom of the Chord, and where the Ruler crosseth the Parallel of 50 degr. as at F, there is your other Place upon the Chart. So E and F lie in 50 degr. of Latitude, and differ in Longitude $5\frac{1}{2}$ degr.

But if the two Places to be set upon the Chart differ onely in Latitude, and lie under the same Meridian as G F, then the Course leading from the one to the other is directly North or South, and the Difference of Latitude of F and G is 2 degr. G lying in the Latitude of 48 degr. and F in the Latitude of 50 degr.

But if the Places to be set upon the Chart differ both in Longitude and Latitude, as A and F, then the Course leading from the one to the other is upon some other Point of the Compass, so far distant from the Meridian as is the quantity of the Angle E A F, which here is 70 degr. 1 min. that is upon the E. N. E. Point 2 degr. 31 min. Easterly.

This Angle may be found either by Protraction by your Line of Chords, or it may be protracted by your *Protracting*

Quadrant, which in all theſe Operations upon the Chart is beſt, for that it avoids the drawing of Arches of Circles upon your Chart or Blank. So then if you were to protract the Angle E A F by your *Protracting Quadrant*; Lay the Centre A of your Quadrant upon the Point A in your Chart, and the Meridian Line of the Quadrant A B upon the Meridian Line of your Chart; then will the Line A C of the



Quadrant lie upon the Parallel A D of your Chart: and the Angle that you are to protract being 70 degr. 1 min. by the edge of your Quadrant make a ſmall Mark or Prick with your Needle, and from A through that Point draw a right Line, which will be the Line A F.

And in the ſame manner as you ſet any Place upon your Chart, you may find in what Latitudes and difference of Longitude any Places already ſet upon your Chart are in.

PROBL. II.

Any Places being ſet upon the Chart, to find in what Latitudes they are, and alſo how they differ in Longitude.

LET the Points Q and R upon the Chart be two Places, and I would know in what Latitudes they lie. Firſt, through the Point Q draw a Line parallel to the Line B C of your Chart; and alſo through the Point R draw another Line parallel to A C. The Line that is drawn through Q ſhoots

ſhoots upon the Latitude of 58 degr. 36 min. and the Line paſſing through R cuts the Meridian of the Chart on either ſide at the Latitude of 57 degr. 16 min. And under thoſe two Latitudes are the two Places Q and R.

Then to find their Difference of Longitude; Take in your Compaſſes the Diſtance between R and S, and meaſuring it upon the bottom of the Chart, it will reach from A to 4 degr. 24 min. And ſuch is the Difference of Longitude of the two Places Q and R.

PROBL. III.

Having the Rhumb, and the Diſtance that the Ship hath run upon that Rhumb, to find the Difference of Longitude and Latitude.

The Analogie or Proportion.

As the Radius is to the Diſtance run;
So is the Sine of the Rhumb to the Difference of Longitude:
And
So is the Co-ſine of the Rhumb to the Difference of Latitude.

So the Rhumb being 70 degr. 1 min. that is E. N. E. 2 degr. 31 min. Eaſterly, and the Diſtance run 117 Leagues, the Difference of Longitude will be found to be 5 ½ degr. and the Difference of Latitude 2 degr.

Upon the Chart.

UPON the Point A protract an Angle of 70 d. 1 m. as the Angle E A F, and draw the Line A F, which is the Rhumb upon which the Ship ſailed. Upon this Line ſet 117, the number of Leagues that the Ship ſailed from A to F. Then through the Point F draw the Line F E parallel to A D. So ſhall E F be the Difference of Longitude, 5 d. and an half, and A E the Difference of Latitude, 2 degr.

PROBL. IV.

The Difference of Latitude and the Rhumb being given, to find the Distance run and the Difference of Longitude.

The Analogie or Proportion.

As the Co-ſine of the Rhumb is to the Difference of Latitude;
So is the Radius to the Diſtance run:
And
So is the Sine of the Rhumb to the Difference of Longitude.

So the one Latitude being 48 degr. and the other 50 degr. the Difference is 2 degr. and the Rhumb being E. N. E. 2 deg. 31 min. Eaſterly, the Diſtance run will be found to be 117 Leagues, and the Difference of Longitude 5½ degr.

Upon the Chart.

SET the Difference of Latitude 2 degr. from A to E, and draw the Line E F parallel to A D. Then upon the Point A protract the Angle of the Rhumb 70 degr. 1 min. E. N. E. 2 degr. 31 min. Eaſterly, and draw the Line A F, cutting the other Line E F in F. Then taking in your Compaſſes the length of the Line A F, and meaſuring it upon the ſide of the Chart, you ſhall find it to contain 117; which is the number of Leagues the Ship ſailed: And the Line E F, being ſo meaſured, will contain 5½ degr. the Difference of Longitude.

PROBL.

Probl. V.

Having the Difference of Longitude and the Rhumb given, to find the Distance run and Difference of Latitude.

The Analogie or Proportion.

As the Sine of the Rhumb is to the Difference of Longitude;
So is the Radius to the Distance run:
And
So is the Co-sine of the Rhumb to the Difference of Latitude.

So the Rhumb being E. N. E. 2 degr. 31 min. Easterly, and the Difference of Longitude $5\frac{1}{2}$ degr. the Distance run will be found to be 117 Leagues, and the Difference of Latitude 2 degr.

Upon the Chart.

UPON the Point A protract an Angle of the Rhumb 70 degr. 1 min. and draw the Line A F. Then the Difference of Longitude being $5\frac{1}{2}$ degr. count $5\frac{1}{2}$ degr. upon the bottom of your Chart from A to G, and upon the Point G raise a Perpendicular G F, cutting the Line A F before drawn in F. Then the Line A F, being measured upon the Side of your Chart, will be found to contain 117 Leagues, the Distance run: And F G, there also measured, will be found to be 2 degr. the Difference of Latitude.

Probl. VI.

The Distance that the Ship hath run, and the Difference of Latitude, given, to find the Rhumb and Difference of Longitude.

The Analogie or Proportion.

As the Distance run is to the Radius ;
So is the Difference of Latitude to the Co-sine of the Rhumb:
And
So is the Sine of the Rhumb to the Difference of Longitude.

So the distance run being 117 Leagues, and the Difference of Latitude being 2 degr. the Rhumb will be found to be E. N. E. 2 degr. 31 min. Easterly, and the Difference of Longitude 5 ½ degrees.

Upon the Chart.

SET the Difference of Latitude 2 degr. upon your Chart from A to E, and draw the Line E F parallel to A B. Then out of the Side of your Chart take the Distance run, 117 Leagues; and setting one foot of the Compasses in A, turn the other about till it cross the Line E F, which it will doe in F. Then F E, being measured upon the bottome of your Chart, will contain 5 ½ degr. the Difference of Longitude. And by your Line of Chords or Protracting Quadrant find the Quantity of the Angle E A F, which will be 70 degr. 1 min. the E. N. E. Point 2 degr. 31 min. Easterly.

Probl.

PROBL. VII.

The Distance that the Ship hath run, and the Difference of Longitude, being given, to find the Rhumb and Difference of Latitude.

The Analogie or Proportion.

As the Distance run is to the Radius;
So is the Difference of Longitude to the Rhumb:
And
So is the Co-sine of the Rhumb to the Difference of Latitude.

So the Difference of Longitude being 5 ½ degr. and the Distance that the Ship hath run 117 Leagues; the Rhumb will be found to be E. N. E. 2 degr. 31 min. Easterly, and the Difference of Latitude 2 degr.

Upon the Chart.

COUNT the Difference of Longitude upon the bottome of the Chart from A to G, and upon the Point G raise the Perpendicular, G F. Then take out of the Side of your Chart the Distance run, 117 Leagues, and setting one foot of the Compasses in A, with the other cross the Perpendicular F G in the Point F. Now if you take F G in your Compasses, and measure it on the Side of your Chart, you shall find it to contain 2 degr. for the Difference of Latitude; and the Angle E A F, being measured by your Chord or Quadrant, will be 70 degr. 1 min. that is the E. N. E. Point 2 d. 31 m. Easterly for the Rhumb.

PROBL. VIII.

The Difference of Longitude and Difference of Latitude being given, to find the Rhumb and the Distance run.

The Analogie or Proportion.

As the Difference of Latitude is to the Radius;
So is the Difference of Longitude to the Tangent of the Rhumb:

And

As the Sine of the Rhumb is to the Difference of Longitude;
So is the Radius to the Distance run.

So the Difference of Longitude being 5 ½ degr. and the Difference of Latitude 2 degr. the Rhumb will be found to be E. N. E. 2 degr. 31 min. Easterly, and the Distance upon the Rhumb 117 Leagues.

Upon the Chart.

COUNT the Difference of Latitude from A to E, and draw the Line E F parallel to A D. Also count the Difference of Longitude from A to G, and upon the Point G raise the Perpendicular G F, cutting the Line E F in the Point F. Then take in your Compasses the length of the Line A F, and measuring it upon the Side of the Chart, you shall find it to contain 117 Leagues, the Distance that the Ship hath run. And if by your Line of Chords, or Quadrant, you find the Quantity of the Angle E A F, it will be the Rhumb, which you may find to be E. N. E. 2 degr. 31 min. Easterly, or 70 degr. 1 min.

PROBL.

PROBL. IX.

The Rhumb that a Ship hath ſailed upon, and the number of Leagues ſhe hath ſailed upon that Rhumb, being given, to know how much ſhe hath raiſed or depreſſed the Pole.

The Analogie or Proportion.

As the Radius is to the Diſtance run;
So is the Co-ſine of the Rhumb from the Meridian to the Difference of both Latitudes.

So the Rhumb being E. N. E. 2 degr. 31 min. Eaſterly, that is, 70 degr. 1 min. and the Diſtance that the Ship hath ſailed upon that Rhumb 117 Leagues, the Pole will be found to be raiſed 2 degr.

Upon the Chart.

UPON the Point A, the leſſer Latitude, protract an Angle of 70 degr. 1 min. and draw the Line of the Rhumb A F, and out of the Side of your Chart take 117 Leagues, (the Diſtance the Ship ſailed) and ſet them upon the Rhumb from A to F. Then through the Point F draw the Line E F parallel to A D, cutting the Meridian of your Chart in E, which is 2 degr. from A: ſo that the Ship hath raiſed the Pole 2 degrees.

Probl. X.

The Longitude and Latitude of the Place from whence you came, with the Rhumb and Distance sailed, being given, to find the Longitude and Latitude of the Place to which you are come.

The Analogie or Proportion.

As the Radius is to the Distance run;
So is the Sine of the Rhumb from the Meridian to the Difference of Longitude:

And

So is the Co-sine of the Rhumb to the Difference of Latitude.

So the Latitude of the Place from whence you came being 52 degr. and the Longitude 35 degr. the Rhumb upon which you have sailed N.E. by N. 33 degr. 45 min. and the Distance which you have sailed upon that Rhumb 96 $\frac{2}{10}$ Leagues; you shall find the Difference of Longitude to be 2 degr. 40 min. and the Difference of Latitude 4 degr. So that the Place to which you are come is in the Latitude of 56 degr. and in the Longitude of 37 degr. 40 min.

Upon the Chart.

THE Place from whence you came being in the Latitude of 52 degr. and in the Longitude of 35 degr. is represented by H. The Rhumb you have sailed upon being N. E. by N. 33 degr. 45 min. upon the Point H protract an Angle of 33 degr. 45 min. and draw the Line H K for the Rhumb. Then out of the Side of your Chart take 96 $\frac{2}{10}$ Leagues, which is so much as the Ship sailed, and set that upon the Rhumb-Line from H to K, and through the Point K draw the Line K L parallel

parallel to B C, (or perpendicular to A B,) and it will cut the Line A B in L. So K L, being meaſured on the bottom of your Chart, will be found to contain 2 degr. 40 min. the Difference of Longitude; which added to 35 degr. the Longitude you came from, gives 37 degr. 40 min. for the Latitude you are in. Alſo the Line H L, being meaſured on the Side of your Chart, will be found to contain 4 degr. And ſuch is the Difference of Latitude, which added to 52 degr. the Latitude from whence you came, gives 56 degr. the Latitude in which you are.

PROBL. XI.

The Longitude and Latitude of the Place from whence you came, the Rhumb upon which you ſailed, and the Latitude of the Place to which you are come, being given, to find the Diſtance and Difference of Longitude.

The Analogie or Proportion.

As the Difference of Latitude is to the Radius;
So is the Tangent of the Rhumb from the Meridian to the Difference of Longitude:

And

As the Sine of the Rhumb is to the Difference of Longitude;
So is the Radius to the Diſtance run.

So the Latitude of the Place from whence you came being 52 degr. and the Longitude 35 degr. and the Rhumb upon which you ſailed the third from the Meridian N. E. by N. 33 degr. 45 min. you ſhall find the Diſtance run to be 96 Leagues $\frac{2}{10}$, and the Difference of Longitude 2 degr. 40 min.

PROBL. X.

The Longitude and Latitude of the Place from whence you came, with the Rhumb and Distance sailed, being given, to find the Longitude and Latitude of the Place to which you are come.

The Analogie or Proportion.

As the Radius is to the Distance run;
So is the Sine of the Rhumb from the Meridian to the Difference of Longitude:
And
So is the Co-sine of the Rhumb to the Difference of Latitude.

So the Latitude of the Place from whence you came being 52 degr. and the Longitude 35 degr. the Rhumb upon which you have sailed N. E. by N. 33 degr. 45 min. and the Distance which you have sailed upon that Rhumb 96 $\frac{2}{10}$ Leagues; you shall find the Difference of Longitude to be 2 degr. 40 min. and the Difference of Latitude 4 degr. So that the Place to which you are come is in the Latitude of 56 degr. and in the Longitude of 37 degr. 40 min.

Upon the Chart.

THE Place from whence you came being in the Latitude of 52 degr. and in the Longitude of 35 degr. is represented by H. The Rhumb you have sailed upon being N. E. by N. 33 degr. 45 min. upon the Point H protract an Angle of 33 degr. 45 min. and draw the Line H K for the Rhumb. Then out of the Side of your Chart take 96 $\frac{2}{10}$ Leagues, which is so much as the Ship sailed, and set that upon the Rhumb-Line from H to K, and through the Point K draw the Line K L parallel

parallel to B C, (or perpendicular to A B,) and it will cut the Line A B in L. So K L, being meaſured on the bottom of your Chart, will be found to contain 2 degr. 40 min. the Difference of Longitude; which added to 35 degr. the Longitude you came from, gives 37 degr. 40 min. for the Latitude you are in. Alſo the Line H L, being meaſured on the Side of your Chart, will be found to contain 4 degr. And ſuch is the Difference of Latitude, which added to 52 degr. the Latitude from whence you came, gives 56 degr. the Latitude in which you are.

PROBL. XI.

The Longitude and Latitude of the Place from whence you came, the Rhumb upon which you ſailed, and the Latitude of the Place to which you are come, being given, to find the Diſtance and Difference of Longitude.

The Analogie or Proportion.

As the Difference of Latitude is to the Radius;
So is the Tangent of the Rhumb from the Meridian to the Difference of Longitude:

And

As the Sine of the Rhumb is to the Difference of Longitude;
So is the Radius to the Diſtance run.

So the Latitude of the Place from whence you came being 52 degr. and the Longitude 35 degr. and the Rhumb upon which you ſailed the third from the Meridian N. E. by N. 33 degr. 45 min. you ſhall find the Diſtance run to be 96 Leagues $\frac{2}{10}$, and the Difference of Longitude 2 degr. 40 min.

Upon the Chart.

UPON your Chart assign H for your Place from whence you came, in the Latitude of 52 degr. and Longitude 35 degr. Upon this Point H protract the Angle of the Rhumb 33 d. 45 m. N.E. by N. and draw the Rhumb-Line H K. Then the Latitude of the Place where you are being found by observation (or being otherwise given) to be 56 degr. draw a Line quite cross your Chart at the 56th degreee of Latitude, as the Line 56. 56 in the Chart crossing the Rhumb-Line in the Point K. So K L, being measured at the bottome of your Chart, will be found to contain 2 degr. 45 min. which added to 35 degr. the Longitude you came from, makes 37 degr. 40 min. And that is the Longitude in which you are. In like manner measure H K upon the Side of your Chart, and you shall find it to contain $96\frac{1}{10}$ Leagues. And so much hath the Ship run upon that Point N. E. by N.

PROBL. XII.

The Latitude of two Places, and the Difference of Longitude between them, being known, to find what Rhumb leadeth from one to the other, and how many Leagues distant they are asunder.

The Analogie or Proportion.

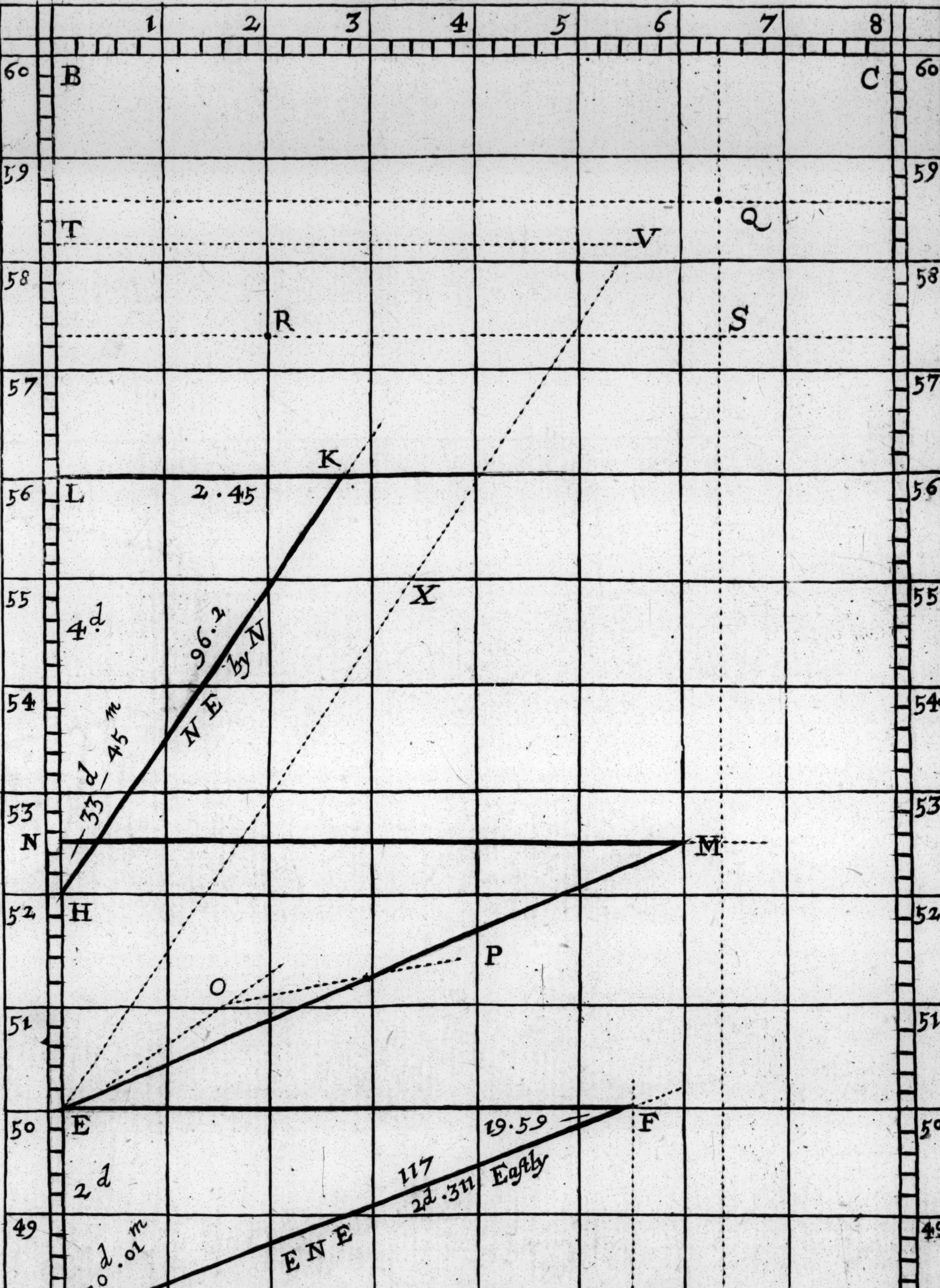
As the Difference of Latitude is to the Radius;
So is the Difference of Longitude to the Tangent of the Rhumb:

And

As the Sine of the Rhumb is to the Difference of Longitude;
So is the Radius to the Distance of the two Places.

So

The Plaine Sea Chart.



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So the Latitude of one of the Places being 50 degr. and the other 52 degr. 30 min. and the Difference of Longitude 6 ½ degrees; the Rhumb will be found to be 67 degr. 23 min. and the Distance upon the Rhumb 6 ½ degr. or 120 Leagues.

Upon the Chart.

UPON the Point of the greater Latitude at N 52 deg. 30 min. draw a Line N M, parallel to A D, upon which Line set 6 degr. the Difference of Longitude of the two Places (being taken from the bottom of the Chart) from N to M. Then from the Point M draw the Line to E, the lesser Latitude, 52 degr. which Line, taken in the Compasses and measured upon the Side of the Chart, will be found to contain 6 ½ degr. or 130 Leagues. Also the Angle N E M, being measured by your Chord, or Protracting Quadrant, will be found to contain 67 degr. 23 min. which is the Rhumb leading from one to the other, namely, short of the E. N. E. Point 7 degr. or, N. E. by E. 11 degr. 8 min. Easterly.

PROBLEMS
Of Sailing by
Mercator's Chart.

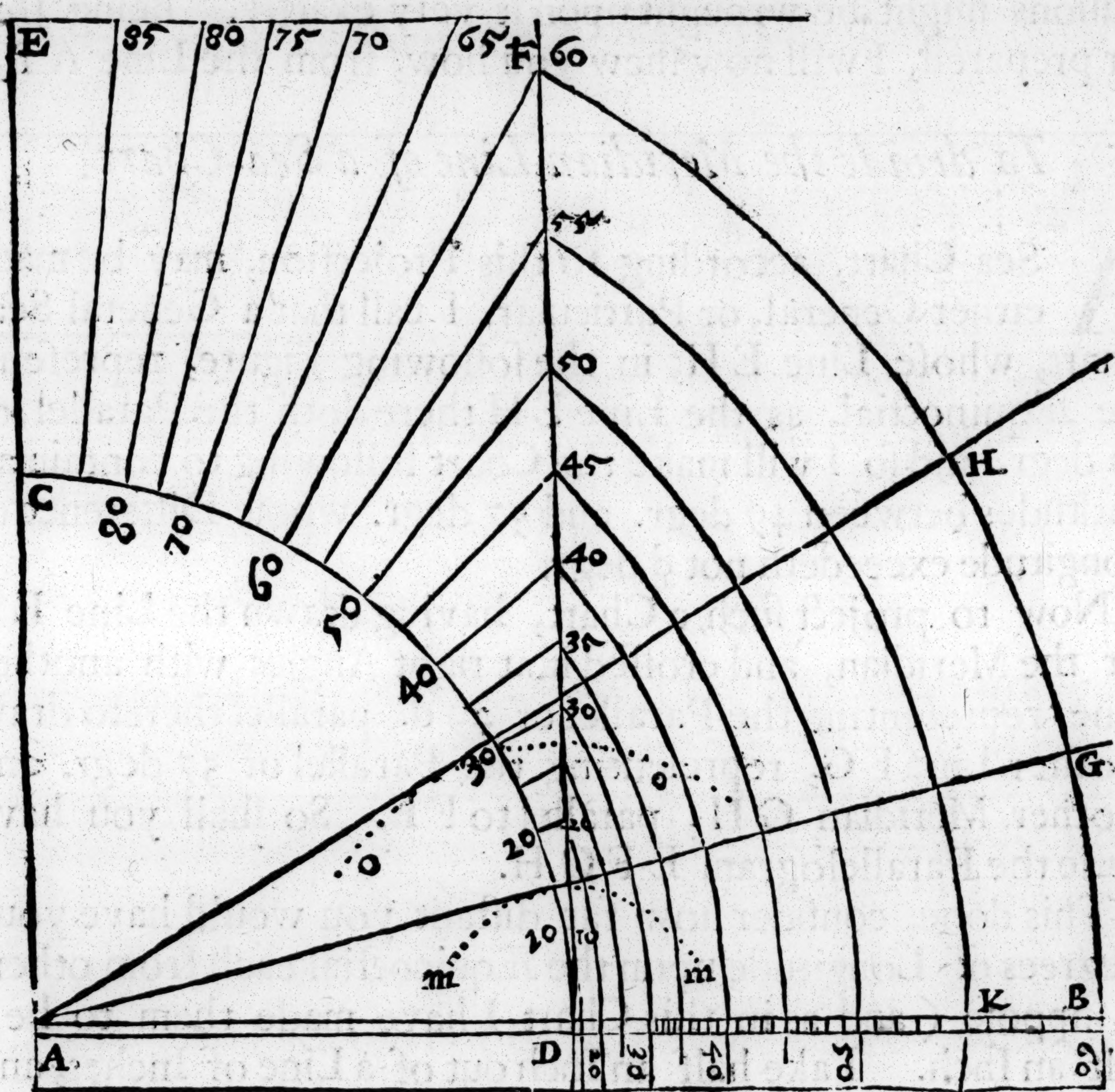
Section III.

Probl. I.

How to make a Sea-Chart according to MERCATOR's Projection, by your Line of Chords.

UPON a piece of thick and smooth Paper (or rather Past-board) draw a right Line, as A B, and upon the Point A, with 60 degr. of your Line of Chords, describe the Quadrant A C D, which divide into 90 equal Parts or Degrees, as here in this Figure there is onely every fifth Degree. ——This done, upon the Point D erect the Perpendicular D F, (in which you must be very exact.) And from the Point A (through each Degree of the Quadrant) draw right Lines, as A 10, 10; A 20, 20; A 30, 30; A 40, 40; &c. towards 90. till they touch the Line D F.——Then with your Com-

Compaſſes, one foot being placed in A, extend the other to 60 degr. in the Line D F, and with that Diſtance deſcribe the Arch F H G B. Again, extend the Compaſſes from A to 55 in the Line F D, and keeping one foot in A, with the other deſcribe 55 K; ſo ſhall the Point K be the Point of 55 degr. in the Line A B.—Alſo, extend the Compaſſes from A to 50 in the Line F D, and draw the Arch 50, 50. Doe ſo with



45, 40, 35, &c. till you come to the beginning of the Degrees at D. So ſhall theſe Arch-lines, by meeting with the *Line* A B, divide that part of it D B into unequal parts, at 10, 20, 30, 40, 50, 60, and ſo forward. But this Figure is ſufficient for Example.

Now

Now from this Line A B, being thus unequally divided, you may divide the Meridian-line of a Sea-Chart according to *Mercator*'s Projection of any bigness, so that the Distance between Degree and Degree in the Æquinoctial be less then the Distance A D, which is here two Inches. And if a Chart were made that the Æquinoctial Degrees were two Inches distant, and it passed upon a smooth Board, many Nauticall Conclusions might be wrought upon it very exactly. Being thus far prepared, I will now shew you how, from the Line A B,

To divide the Meridian Line of a Sea-Chart.

A Sea-Chart, according to this Projection, may be made either General, or Particular. I call that a General Sea-Chart, whose Line E H, in the following Figure, represents the Æquinoctial, as the Line E H there doth the Parallel of 49 degr. and so I will make the Chart following to contain all Latitudes between 49 degr. and 57 degr. whose Difference of Longitude exceedeth not 8 degr.

Now to project such a Chart, having drawn the Line E F for the Meridian, and crossed it at right Angles with another Line representing the Parallel of 49 d. parallel thereto draw another Line F G, representing the Parallel of 57 degr. and another Meridian G H, parallel to F E. So shall you have made the Parallelogram E F G H.

This done, consider how far distant you would have your Degrees of Longitude upon the Æquinoctial each from other, as suppose (and as in this Chart I have made them to be) half an Inch. Take half an Inch out of a Line of Inches, and run that Distance along the Line E H from E to 1, from 1 to 2, from 2 to 3, &c. And also doe the like upon the Line F G, at the top of the Chart, drawing the Lines 1, 1; 2, 2; 3,3; &c.

Now for the Dividing of the Meridians E F and H G, repair to the foregoing Figure, taking in your Compasses the Distance that is between Degree and Degree of the Æquinoctial,

ctial, which in our Example is half an Inch. With this Distance, set one foot of the Compasses in the Point D, and with the other describe the Arch *m m*; by the very Edge whereof draw the Line A G: so is your Figure prepared to divide the Meridian-line of a Sea-Chart whose Degrees of Longitude are half an Inch distant.

Now in respect that your first Parallel of Latitude E H in your Chart is drawn for 49 degr. your next Parallel must be 50 degr. Wherefore set one foot of your Compasses upon 50 degr. in the Line A B, and with the other take the nearest Distance to the Line A G: that is done by turning the Compasses about till the moveable foot do onely touch the Line A G; which when it so doth, that Distance at which your Compasses then are, being set upon the Meridian of your Chart, will reach from 49 degr. to 50, which being set upon your Chart, on both sides thereof, from 49 draw the Line 50. 50 will give you the Parallel of 50 d. of Latitude. In like manner for the Parallel of 51 degr. Set one foot of the Compasses in 51 degr. upon the Line A B of the former Figure, and with the other take the least Distance to the Line A G: this Distance set upon the Meridian of your Sea-Chart, on both sides thereof, will reach from 50 to 51; and there draw the Parallel 51, 51.—— Likewise for the Parallel of 52 degr. Set one foot of the Compasses in 52 degr. in the Line A B, taking the nearest Distance to the Line A G: that Distance set upon the Meridian of your Sea-Chart, on both sides thereof, will reach from 51 to 52; and there draw the Parallel of 52, 52. Doe thus with all the Degrees, as 53, 54, 55, 56, and 57. So shall the Meridians of your Chart E F and H G be divided into whole Degrees.

For the Sub-divisions of these Degrees, they may be divided each of them into equal parts, as the Divisions at the top and bottome of the Chart ought to be; but the Degrees of the Meridian, as they grow higher, they ought still to grow greater. But the Difference is so small, that it cannot produce

any considerable Errour, though the Sub-divisions be all made equal between Degree and Degree. You may therefore divide them either into 60 Minutes or English Miles, or into 20 Leagues, or into 100 parts of Degrees, as you shall best like of.

But if you would make a Chart that the Distance between Degree and Degree upon the Æquinoctial should be an Inch, or any other Distance less then A D in the foregoing Figure; take that Distance (as suppose an Inch) in your Compasses, and setting one foot in D, with the other describe the Arch o o, and draw the Line A H onely to touch the Arch o o. The least Distance taken from each Degree to this Line A H shall give you the Distance of the Degrees upon the Meridian of a Sea-Chart, whose Distance of Degrees upon the Æquinoctial are an Inch from each other.

Your Chart being thus prepared, I will now come to shew you how to resolve severall Problems upon it.

Probl. II.

To find how many Leagues do answer to one Degree of Longitude in every severall Latitude.

UPON the two edges of your Protracting Quadrant there are two Lines, the one divided into 20, the other into 60 equal parts.

Take therefore the least Distance from the Complement of the Parallel's distance from the Æquator, (or the Complement of the given Latitude:) this Distance, being measured upon the edge that is divided into 20, shall shew you what number of Leagues make one Degree of Longitude in that Parallel of Latitude. And the same Distance, being measured upon the other edge that is divided into 60, will give so many of our Miles, or so many Minutes of the Æquinoctial, or any other great

great Circle, as are anſwerable to one Degree of Longitude in that Latitude.

Example. Let it be required to find how many Leagues do anſwer to one Degree of Longitude in the Latitude of 18 d. 12 min.

Set one foot of your Compaſſes in 71 degr. 48 min. the Complement of the given Latitude, and with the other take the neareſt Diſtance to the ſide of the Quadrant which is divided into 20: that Diſtance, meaſured upon the Line 20, will reach from the beginning thereof to 19: and ſo many Leagues do anſwer to one Degree of Longitude in the Latitude of 18 degr. 12 min.

Or, If you take the leaſt Diſtance from 18 degr. 12 m. the Latitude it ſelf, in the Limb of the Quadrant, to that edge which is divided into 60, that Diſtance will alſo reach to 19 upon the Line 20, as before.

And the ſame Diſtance, being meaſured upon the Line 60 of the Quadrant, will give you 57 parts: and ſo many Minutes of the Æquator are anſwerable to one Degree of Longitude in the Parallel of 18 degr. 12 min. of Latitude.

So likewiſe in the Latitude of 25 degr. 15 min. if you take the leaſt Diſtance from the Complement thereof, or from the Latitude it ſelf, to the edges of the Quadrant, you ſhall find that Diſtance to reach 18 in the Line of 20: and ſo many Leagues do anſwer to one Degree of Longitude in the Latitude of 25 degr. 15 min. or unto 54 in the Line of 60: and ſo many Minutes of the Æquator do anſwer to one Degree of Longitude in that Parallel of Latitude.

The Analogie or Proportion.

As the Radius is to the Co-ſine of the Latitude;

So is $\left\{\begin{matrix}\text{20 Leagues}\\\text{60 Minutes}\end{matrix}\right\}$ to the number of $\left\{\begin{matrix}\text{Leagues}\\\text{Minutes}\end{matrix}\right\}$ which anſwer to one Degree of Longitude in that Latitude.

Probl. III.

By the Latitude of two Places and their Distance, to find the Rhumb.

The Analogie or Proportion.

As the Distance upon the Rhumb is to the Radius;
So is the Difference of Latitudes to the Co-sine of the Rhumb from the Meridian.

Thus if the Places given were one in the Latitude of 50 d. and the other in the Latitude of 55 degr. and the Distance upon the Rhumb 6 degr. or 120 Leagues; the Rhumb leading from one to the other will be found to be the third from the Meridian, namely, N. E. by N. 33 degr. 45 min.

Upon the Chart.

LET A represent the Place in the Latitude of 50 degr. and C that in 55 degr. whose Distance from A to C is 6 degr. Take 6 degr. out of the Meridian-line, by setting one foot as much below the lesser Latitude as above the greater, which will be from K in the Latitude of 49 ½ degr. to L in the Latitude of 55 ½; either of which are half a Degree above and under the two given Latitudes. Take this Distance K L in your Compasses, and setting one foot in A, (the lesser Latitude) with the other cross the Parallel of the greater Latitude 55 degr. in the Point C, and draw a right Line from A to C. So shall the quantity of the Angle B A C, being found (either by your Chord or Quadrant,) shew you the Inclination of the Rhumb to the Meridian to be 33 degr. 45 min. the N. E. by N. Point.

Note, *That in the Propositions following, the Difference of Longitude must always be taken out of the Æquator, and measured there-*

thereupon also. But the Difference of Longitude and Distance upon the Rhumb must alwaies be measured upon, and taken out of, the Meridian Line of your Chart. And hereafter I shall call them the proper Difference, and proper Distance.

PROBL. IV.

The Longitude and Latitude of two Places being given, to find the Rhumb.

The Analogie or Proportion.

As the proper Difference of Latitude is to the Radius;
So is the Difference of Longitude to the Tangent of the Rhumb from the Meridian.

Thus if the Places should lie one in the Latitude of 50 deg. and the other in the Latitude of 55 degr. and the Difference of Longitude between them were 5 degr. 30 min. the Rhumb leading from one Place to the other will be found to be the third from the Meridian N. E. by N. 33 degr. 45 min.

Upon the Chart.

THE Meridians and Parallels being drawn through the two Places at A and C, and a straight Line from A to C, for the Rhumb, by your Chord or Quadrant find the quantity of the Angle B A C, which you will find to be 33 d. 45 m. or the third Rhumb from the Meridian N. E. by N.

But if this Rhumb were to be found by the Common Sea-Chart, it would be found to be above 47 degr. that is, N. E. 2 degr. Easterly, that is, one whole Point and 2 degr. more Easterly then it should be.

PROBL. V.

The Latitude of two Places and the Rhumb being given, to find the Difference of Longitude.

The Analogie or Proportion.

As the Radius is to the Tangent of the Rhumb from the Meridian;
So is the proper Difference of Latitudes to the Difference of Longitude.

Thus the Latitude of one Place being 50 degr. and the other 55 degr. and the Rhumb leading from one to the other being the third from the Meridian, the Difference of Longitude will be found to be 5 ½ degr.

Upon the Chart.

LET a Meridian be drawn through A, and a Parallel of Latitude through C. Then upon the Angle A protract the Angle of the Rhumb 33 degr. 45 min. So the Distance B C upon the Parallel, being measured upon the bottome of the Chart, will be found to contain 6 degr. 30 min.

But if this Difference of Longitude were to be found by the Plain Sea-Chart, the Difference of Longitude would be found to be but 3 *degr.* 20 *min. which is more then* 3 *degr. less then the truth; a vast Difference. And yet this Errour would be yet greater, if either the Latitude be greater, or the Rhumb farther from the Meridian.*

Probl. VI.

The Difference of Longitude of two Places, the Latitude of one of them, and the Rhumb leading from one to the other, given, to find the Latitude of the other Place.

The Analogie or Proportion.

As the Radius is to the Co-tangent of the Rhumb from the Meridian;
So is the Difference of Longitude to the proper Difference of Latitude.

Thus if the Latitude of one of the Places were 50 degr. the Rhumb leading from that to the other N. E. by N. 33 d. 45 min. and the Difference of Longitude between the two Places were 5 degr. 30 min. the Latitude of the other Place will be found to be in 55 degr.

Upon the Chart.

LET AB and DC be two Meridians drawn through A and C, at 5 ½ d. the Difference of Longitude, and a Parallel of Latitude through A, crossing the Meridian CD in D. Then upon the Point A protract an Angle equal to the Rhumb from the Meridian given 33 degr. 45 min. So the Line CD, being measured upon the Meridian from A, the given Latitude, 50 degr. will reach to 56 degr. the proper Difference of Latitude. So that the other Place lies in the Latitude of 56 degr.

But if this Difference of Latitude were to be found by the Plain Sea-Chart, this Difference of Latitude would be found to be 8 *d.* 13 *min. and the Latitude sought would be found to be* 58 *degr.* 13 *min.*

13 min. above three Degrees more then the truth. As by the Triangle for that purpose drawn upon the Plain Sea-Chart, marked with T V E, may appear.

Probl. VII.

Having the Latitude of one Place, the Rhumb leading from that Place to another unknown, and the Distance upon the Rhumb from the first to the second Place, to find the Difference of Longitude of the two Places.

The Analogie or Proportion.

As the Radius is to the Sine of the Rhumb from the Meridian; So is the proper Distance upon the Rhumb to the Difference of Longitude.

Thus if the two Places were one in the Latitude of 50 degr. and the other in a greater Latitude, but unknown; the proper Distance upon the Rhumb leading from one place to the other being 6 degr. and the Rhumb N. E. by N. 33 degr. 45 min. the Difference of Longitude will be found to be 5 ½ degr.

Upon the Chart.

THrough the Point A in the Latitude of 50 degr. let be drawn a Meridian A B, and a Parallell A D; and upon the Point A protract an Angle equal to the Rhumb from the Meridian 33 degr. 45 min. Then take with the Compasses 6 degres, the proper Distance upon the Rhumb, out of the Meridian-line, (having respect to the Latitude of the Places) as from K to L, and set that Distance upon the Rhumb from A to C. Then through C draw another Meridian C D, crossing the Parallel drawn through A in the Point D. So the Line A D, being measured at the bottom of the Chart, will be found to contain 5 ½ d. the Difference of Longitude sought.

But

But if this Difference of Longitude had been to be found by the Common Sea-Chart, it would be found to have been onely 3 *d.* 20 *min. which is* 2 *degr.* 10 *min. less then the truth; as in the Plain Chart may be seen, where the third Rhumb from the Meridian cuts the Parallel of* 55 *degr. of Latitude in* 3 *degr.* 20 *m. of Longitude at the Point X.*

PROBL. VIII.

The Difference of Longitude between two Places, the Rhumb leading from one Place to the other, and the Latitude of one of the Places, being given, to find their Distance.

The Analogie or Proportion.

As the Sine of the Rhumb from the Meridian is to the Difference of Longitudes;
So is the Radius to the proper Distance of the two Places upon the Rhumb.

Thus, if the Latitude of one Place were in 50 degr. the other in a greater Latitude unknown, the Difference of Longitude between the two Places $5\frac{1}{2}$ degr. and the Rhumb N. E. by N. 33 degr. 45 min. from the Meridian; the proper Distance upon the Rhumb will be found to be 6 degrees.

Upon the Chart.

LET two Meridians, A B and C D, be drawn through A and C, according to the Difference of Longitude, and a Parallel of Latitude through A, crossing the Meridian C D in the Point D. Then upon the Point A protract an Angle of 33 degr. 45 min. the quantity of the Rhumb from the Meridian, and draw the Line A C crossing the Meridian C D in C. So the Distance C D, being taken in the Compasses, and

measured upon the Meridian-line of the Chart, (respect being had to the Latitude of the Places) that is, so much above the greater Latitude as below the lesser Latitude, you will find it to contain 6 degr.

But if this setting of the Compasses so much above one Latitude as below another seem difficult, it may be thus otherwise done.———For, the Rhumb Line being drawn, it will cut the Meridian C D in C: so a Parallel drawn through C will cut the Meridian A B in B: so is B the Latitude of the second Place, viz. 55 degr. Then divide the Distance between the two Latitudes A and B in two equal parts in the Point M; also divide the Rhumb-Line A C in two equal parts in N: then take the Distance N C or N A, and setting one foot of the Compasses in M, the other will reach to L above the greater Latitude, and from M to K as much below the lesser Latitude, namely, 30 min. or half a Degree on either side; so that between K and L are contained 6 degr. and that is the proper Distance upon the Rhumb.

But if this Distance were to be found by the Plain Chart, it would be almost 10 degr. or 197 Leagues, which is 77 Leagues more then in truth it should be. As may appear, if you measure the Line A L in the Plain Chart, upon the Side thereof.

PROBL. IX.

The Difference of Longitude, and Distance of two Places, with the Latitude of one of the Places, being given, to find the Rhumb that leads from one to the other.

The Analogie or Proportion.

As the proper Distance upon the Rhumb is to the Difference of Longitude;
So is the Radius to the Sine of the Rhumb from the Meridian.

Thus

Thus, if one of the Places lay in the Latitude of 50 degr. and the other in a greater Latitude, but unknown; the Difference of Longitude between them 5 ½ degr. and their proper Diſtance upon the Rhumb 6 degr. the Inclination of the Rhumb to the Meridian which leadeth from one Place to the other will be found to be 33 degr. 45 min. that is the N. E. by N. Point.

Upon the Chart.

LET the Meridians A B and D C be drawn through A and C, and through A a Parallel of Latitude A D. Then open the Compaſſes (having reſpect to the Latitudes) from K to L, the quantity of 6 degr. in the Meridian; and ſetting one foot of that Extent in A, with the other foot croſs the Meridian C D in C, and draw the right Line A C for the Rhumb. Laſtly, by your Chord or Quadrant find the quantity of the Angle B A C, 33 degr. 45 min. and that is the Rhumb required N. E. by N.

But if you were to find this Rhumb by the Plain Sea-Chart, it would be found almoſt the E.N.E. Point within 1 degr. 30 min. differing from truth very near 3 whole Points to the Eaſtward.

PROBL. X.

The Longitude and Latitude of two Places being given, to find the Diſtance upon the Rhumb.

The Analogie or Proportion.

As the proper Difference of Latitudes is to the Radius;
So is the Difference of Longitudes to the Tangent of the Rhumb from the Meridian:

And

As the Sine of the Rhumb from the Meridian is to the Difference of Longitude;

So is the Radius to the proper Distance upon the Rhumb.

Thus, the two Places being one in the Latitude of 50 degr. the other in the Latitude of 55 degr. and the Difference of Longitude between them being $5\frac{1}{2}$ degr. the proper Distance upon the Rhumb will be found to be 6 degr.

Upon the Chart.

DRAW the Meridians A B and C D, the Difference of Longitude between them being $5\frac{1}{2}$ degr. and through A and B draw two Parallels B C and A D, and then the Line for the Rhumb leading from the one to the other A C. So A C, being taken in the Compasses, and measured upon the Meridian-line of the Chart, with this Condition, that at the resting of the Compasses upon the Meridian-line, one foot be so many Degrees above the greater Latitude as the other foot is below the lesser Latitude; so will the feet of the Compasses rest in the Points K and L, one being 30 min. below the lesser Latitude, and the other 30 min. above the greater.

But if this Distance upon the Rhumb were to be found by the Plain Chart, it would be found to be almost 7 degr. 15 min. or 245 Leagues, which is 25 Leagues more then it should be.

PROBL. XI.

The Latitude of two Places and their Distance upon the Rhumb being given, to find their Difference of Longitude.

The Analogie or Proportion.

As the proper Distance upon the Rhumb is to the Radius;
So is the proper Difference of Latitudes to the Co-sine of the Rhumb from the Meridian:

And

And

So is the Sine of the Rhumb from the Meridian to the Difference of Longitude.

Thus, if one of the Places be in the Latitude of 50 degr. and the other in 55 degr. and their proper Distance upon the Rhumb 6 degr. or 120 Leagues; their Difference of Longitude will be found to be 5 degr. 30 min.

Upon the Chart.

DRaw A D and B C, two Parallels of Latitude, through 50 degr. and 55 degr. which were the two given Latitudes. Then out of the Meridian Line take the proper Distance upon the Rhumb (having respect to both Latitudes) from K to L: the Compasses being opened to this Distance, one foot being set in A, the lesser Latitude, the other will cross the Parallel of the greater Latitude in C. So the Distance B C, being measured at the bottome of the Chart from E, will reach to 5 degr. 30 min. And such is the Difference of Longitude between the two Places.

But if this Difference of Longitude were to be found by the Plain Chart, it would be but 3 *degr.* 20 *min. which is no less then* 2 *d.* 10 *min. less then the truth; as by the Triangle* T V E *drawn upon the Plain Chart may appear.*

PROBL. XII.

The Difference of Longitude of two Places, their Distance upon the Rhumb, and the Latitude of one of the Places, being given, to find the Difference of Latitudes.

The Analogie or Proportion.

As the proper Distance of the two Places upon the Rhumb is to the Radius;

So is the Difference of Longitudes to the Inclination of the Rhumb to the Meridian:

And

So is the Co-ſine of the Rhumb from the Meridian to the Difference of Latitudes.

Thus, the Difference of Longitudes being 5 ½ degr. their proper Diſtance upon the Rhumb 6 degr. and the Latitude of one of the Places 50 d. the Difference of Latitudes will be found to be 5 d.

Upon the Chart.

THrough the given Latitude A draw a Meridian AB, and a Parallel A D, and upon the Parallel ſet the Difference of Longitude 5 ½ d. taken from the bottom of the Chart, from A to D, and through D draw the Meridian D C. Then out of the Meridian-line take the proper Diſtance upon the Rhumb, 6 d. from K to L, and ſetting one foot of the Compaſſes in A, with the other croſs the Meridian C D in C: ſo a Parallel of Latitude drawn through C will be the Parallel of 55 d. So is 55 d. the Latitude of the other Place, and 50 being taken from 55, leaves 5 d. for the Difference of Latitudes required.

Which Difference, had it been to be found by the Plain Chart, would have been but 2 d. 25 m. that is, 2 d. 35 m. leſs then the truth; as by the Triangle T V E upon the Plain Chart may appear.

PROBL. XIII.

The Latitude of two Places and their Difference of Longitudes being given, to find the Rhumb leading from one to the other, and alſo how many Degrees diſtant they are aſunder.

THIS Propoſition is already performed in the Example of the two Places A and B; but for Variety I will take two other Places, and onely ſhew the manner of working upon the Chart. Sup-

Suppose then two Places, one (as before) in the Latitude of 50 d. the other in the Latitude of 52 degr. 30 min. whose Difference of Longitudes is 6 degr.

Upon the Chart.

Through the two given Latitudes 50 d. and 52 $\frac{1}{2}$, at A and O draw two Parallels, O P and A D, upon which set the Difference of Longitudes from O to P, and from A to Q, 6 degr. Then draw the Line AP, which shall be the Line of the Rhumb leading from one Place to the other: wherefore, by your Chord or Protracting Quadrant find the quantity of the Angle O A P, which shall be the Inclination of the Rhumb to the Meridian, and will be found to be 56 d. 15 m. that is the N. E. by E. Point; which was the First thing that was required.

Then to find the proper Distance upon the Rhumb; Take the Line A P in your Compasses, and measure it upon the Meridian-line, so that one foot may be above the greater Latitude so much as the other is below the lesser; and you will find the Compass-points to rest in E and S, E being one whole Degree below the lesser Latitude, and S one Degree above the greater. So that there is intercepted between E and S 4 $\frac{1}{2}$ degr. And that is the proper Distance upon the Rhumb; which was the Second thing required.

But if this Problem had been wrought upon the Plain Chart, the Rhumb from the Meridian would be found to be 67 d. 23 m. that is, within 7 m. of the 6th Rhumb; which is more then the truth by 11 d. 8 m.

PROBL. XIV.

A Ship set sail from the Latitude of 50 degr. upon the fifth Rhumb N. E. by E. after that she had made 36 Leagues of way upon that Rhumb, the wind changing, she was constrained to sail 50 Leagues upon the 7th Rhumb E. by N. I would know in what Longitude and Latitude the Ship is.

Upon

Upon the Chart.

THE Rhumb-Line A P being drawn, set off thereupon 36 Leagues (which was the way that the Ship made upon the fifth Rhumb before the Wind changed) from A to T, (which Distance must be taken out of the Meridian-line by opening the Compasses from 50 d. to 51, 48. or better, to as much below 50 d. as above 51 d.) So shall the Point T be the Place that the Ship was in when the Wind altered. So a Parallel drawn through T upon the Chart will cut the Meridian at V in 51 d. and in that Latitude the Ship was. Now to find in what Longitude she was; Take in your Compasses the Line T V, and measure it at the bottom of the Chart, you shall find it will reach from E to 2 d. 21 m. And in that Longitude the Ship then was.

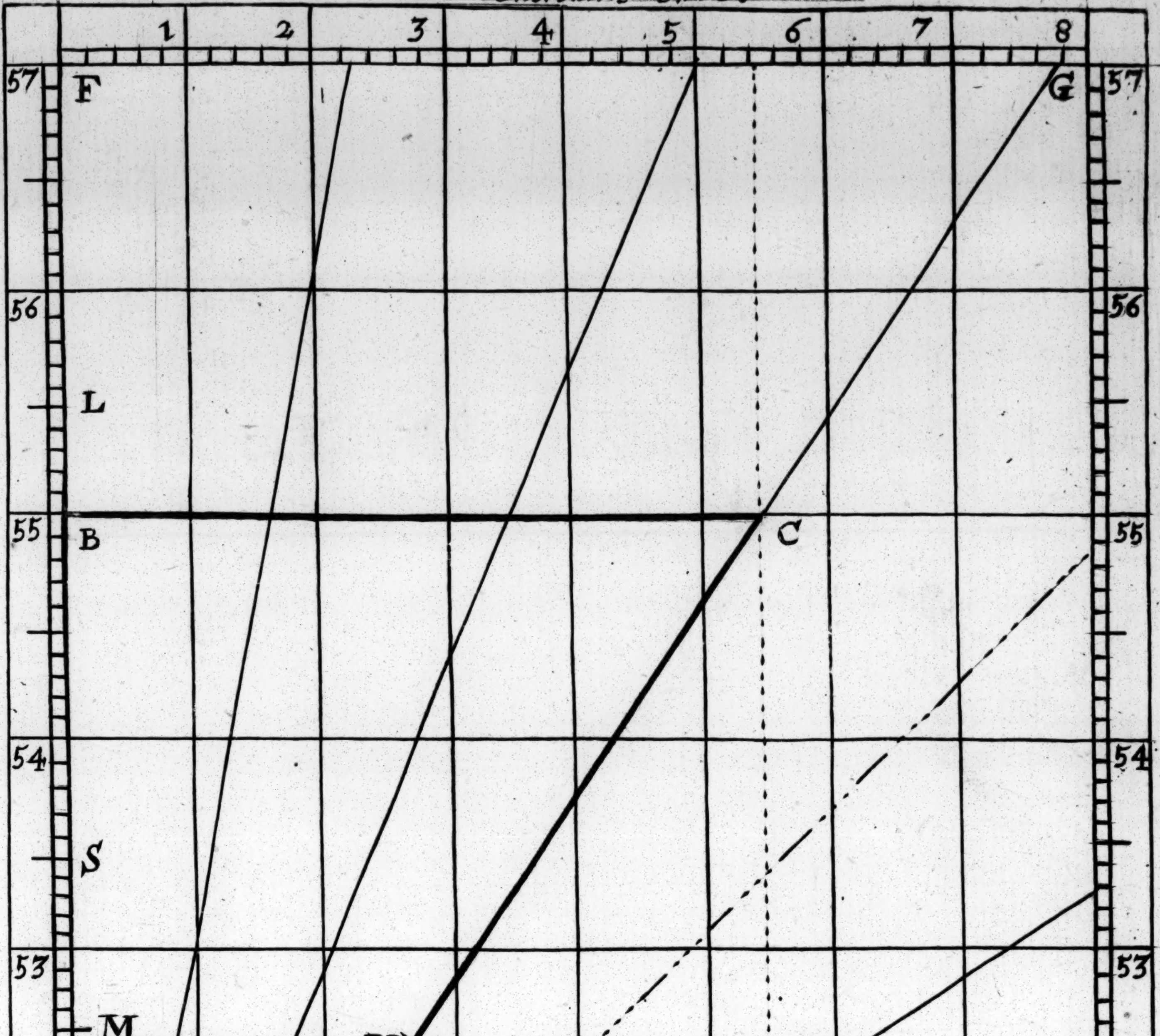
This done, upon the Point T (where the Wind changed, and drove the Ship 2 Points more Eastwardly, namely, upon the E. by N. Point) protract an Angle of 22 d. 30 m. namely, the Angle P T X, which is the Rhumb upon which the Ship sailed 50 Leagues after the Wind changed. Therefore take 50 Leagues out of the Meridian-line, and set them from T to X. So shall X be the Place that the Ship was in after she had sailed 50 Leagues upon the E. by N. Point; which, by drawing a Parallel through K, will be found in the Latitude of 51 d. 30 m. and by drawing of a Meridian through K also, it will be found to be in the Longitude of 6 degr. 16 min.

But if these Courses had been protracted according to the Plain Sea-Chart, the Point T would fall in the Latitude of 51 degr. and the Point X in the Latitude of 51 degr. 30 m. But the Longitude of T would be onely 1 d. 30 m. and the Longitude of X in 3 d. 57 min. Both these Longitudes being added, make but 5 d. 27 m. for the Difference of Longitude between X and the first Meridian; whereas by the other Chart it is 6 d. 16 m. So that the Ship at X is 33 m. Westward of the Place to which she was bound.

These Differences, which I have observed to be between the Plain and Mercator's *Chart, may be seen by comparing the Scheme of the two Charts together.*

F I N I S.

Mercators Chart.



place this at the end of the Booke